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DISTRIBUTION OF LIGHT RADIANCE REFLECTED FROM THE INHOMOGENEOUSLY ILLUMINATED SEA

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ABSTRACT

This work is a continuation and further development of the previously published approaches to solve radiative transfer equation for seawater. The major feature of these approaches is that they do not imply any restriction on values of the inherent optical properties.

The idea of this approach consists of the following: the radiance angular distribution is split into three components: unscattered, single-scattered and multiple-scattered components. Exact solutions for the first two components are found. An approximate solution to the third, multiple-scattered component, is found with the approach derived from the self-consistent theory. Because the major part of radiance distribution is calculated precisely, the resulting precision of final equations exceeds the precision of the self-consistent approach.

Keywords: ocean optics, inhomogeneous illumination, reflection of light

1. INTRODUCTION

Radiance distribution of a sunlight reflected from a homogeneous shallow sea is calculated. The two different cases are considered: a simple homogeneous illumination by sky and sun, and an illumination with a shadow in the form of infinite stripe.

This work is a continuation and further development of the previous approaches published in Refs. [1–7].

In order to maximize the precision of the approach the radiance angular distribution is divided into three components: unscattered, single-scattered and multiple-scattered components. Exact solutions for the first two components are found. An approximate solution to the third, multiple-scattered component, is found with the approach derived from the self-consistent theory [3]. Because the major part of radiance distribution is calculated precisely, the resulting precision of final equations exceeds the precision of the self-consistent approach.

In the last part of this paper a solution to the two-dimensional radiative transfer problem for the sea shadowed by an opaque body is found. The solution to the problem requires two different theoretical approaches. The solution for the depth dependence of light radiance is obtained using the theory of Green's functions. The solution to the problem in horizontal direction is obtained with the one-dimensional Fourier transform.

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2. HOMOGENEOUS SHALLOW SEA ILLUMINATED BY SUNLIGHT

Let us introduce the following notations: E_S is irradiance by the sun on the sea surface, $E_S^0 = E_S / \mu_S^0$ is irradiance by the sun on the surface normal to the sun rays, $\mu_S^0 = \cos Z_S \equiv \sin h_S$ is the cosine of zenith angle Z_S, $h_S = 90° - Z_S$ is a sun elevation angle.

The sun radiance above the sea surface will be

$$L_S^0 = E_S^0 \, \delta(\mu - \mu_S^0)\, \delta(\varphi), \tag{1}$$

here $\delta(x)$ is a Dirac's delta-function [8], $\mu_S = \sqrt{1 - \sin^2 Z_S / n_w^2}$ is a cosine of penetration angle, n_w is the seawater refraction coefficient.

If $T_S^{\downarrow} = 1 - R_F^{\downarrow}(Z_S)$ is a transmission of sea surface, where R_F is a Fresnel reflection coefficient from above, than the radiance below the sea surface is:

$$L_w^0(\mu,\varphi) = E_w^0 \, \delta(\mu - \mu_S)\, \delta(\varphi), \quad E_w^0 = E_S^0 \, T_S^{\downarrow}, \tag{2}$$

here E_w^0 is the irradiance by the sun on the surface normal to the rays just below the sea surface.

The one-dimensional scalar radiative transfer equation for the total radiance is [9, 10]:

$$\left(\mu \frac{d}{dz} + c\right) L(z,\mu,\varphi) = \frac{b}{4\pi} \int_0^{2\pi} d\varphi' \int_{-1}^{1} p(\cos\gamma)\, L(z,\mu',\varphi')\, d\mu', \tag{3}$$

here $c = a + b$ is the attenuation coefficient, a is the absorption coefficient and b is the scattering coefficient of seawater, $\mu = \cos\theta$, the direction of light propagation is determined by the zenith and azimuth angles θ and φ, $p(\cos\gamma)$ is the scattering phase function, γ is the scattering angle determined by $\gamma = \cos^{-1}(\mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos[\varphi - \varphi'])$.

Let us represent total radiance of light inside the sea L as the direct light radiance L_0, and the sum of the radiances L_n that represent n^{th} order of scattering:

$$L = \sum_{n=0}^{\infty} b^n L_n . \tag{4}$$

Substitution of Eq. (5) into Eq. (3) gives us the following equations for radiance components:

$$\left(\mu \frac{d}{dz} + c\right) L_0(z,\mu,\varphi) = 0, \tag{5}$$

$$\begin{aligned} &\hat{D} L_n(z,\mu,\varphi) = b \hat{S} L_{n-1}(z,\mu,\varphi), \quad \text{or} \\ &L_n(z,\mu,\varphi) = b \hat{D}^{-1} \hat{S} L_{n-1}(z,\mu,\varphi) = b \hat{T} L_{n-1}(z,\mu,\varphi). \end{aligned} \tag{6}$$

Operators $\hat{D}$, $\hat{S}$ and $\hat{T}$ in Eq. (6) are defined as follows:

$$\left.\begin{aligned} \hat{D}F(z,\mu,\varphi) &\equiv \left(\mu\frac{d}{dz}+c\right)F(z,\mu,\varphi), \\ \hat{S}F(z,\mu,\varphi) &\equiv \frac{1}{4\pi}\int_0^{2\pi} d\varphi' \int_{-1}^{1} p(\cos\gamma)F(z,\mu',\varphi')\,d\mu', \end{aligned}\right\} \quad \begin{aligned} \hat{D}^{-1} &\equiv \hat{G}, \\ \hat{T} = \hat{D}^{-1}\hat{S} &\equiv \hat{G}\hat{S}. \end{aligned} \tag{7}$$

Operator $\hat{G}$ is a Green's function of Eq. (3). It is defined according to the following:

$$\hat{G}F(z) \equiv \int_{-\infty}^{\infty} G(z-z')F(z')\,dz', \quad G(z) = H(z/\mu)e^{-cz/\mu}/|\mu|, \tag{8}$$

Here $H(z)$ is the Heavyside's or step function defined by $(H(z)=1,\ z\ge 0;\ H(z)=0,\ z<0)$.
Total scattering operator is:

$$\hat{T}F(z,\mu,\varphi) \equiv \frac{1}{4\pi}\int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu'\, p(\cos\gamma) \int_{-\infty}^{+\infty} dz' G(z-z')F(z',\mu',\varphi') \,. \tag{9}$$

Now, the total radiance is expressed as a following sum:

$$L = \sum_{n=0}^{\infty} b^n L_n, \quad L_n = b^n \hat{T}^n L_0,\ n\ge 1,\ L(z,\mu,\varphi) = \sum_{n=0}^{\infty} b^n \hat{T}^n L_0(z,\mu,\varphi). \tag{10}$$

Let us start solving these equations.

3. SOLUTIONS FOR THE RADIANCE

The solution for direct (unscattered) radiance L_0 is straightforwardly obtained from the Eq. (5) with the following boundary condition on the sea surface, $L_0(0,\mu,\varphi) = L_w^0(\mu,\varphi)$:

$$L_0(z,\mu,\varphi) = E_w^0 e^{-cz/\mu_S}\delta(\mu-\mu_S)\delta(\varphi). \tag{11}$$

Equation (6) for the single-scattered radiance can be written as:

$$L_1(z,\mu,\varphi) \equiv \frac{1}{4\pi}\int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu'\, p(\cos\gamma) \int_{-\infty}^{+\infty} dz' G(z-z')L_0(z',\mu',\varphi') \,. \tag{12}$$

By substituting Eq. (11) into Eq. (12), we have the following solution for single-scattered radiance distribution:

$$L_1(z,\mu,\varphi) = \frac{E_w^0\, p(\cos\gamma_S)}{4\pi c|\mu|}\left[\psi_1(z,\mu)H(\mu) + \psi_2(z,\mu)H(-\mu)\right], \tag{13}$$

here

$$\psi_1(z,\mu)=\frac{e^{-cz/\mu_S}-e^{-cz/\mu}}{1/\mu-1/\mu_S},\ \psi_1(z,\mu_S)=c\,z\,e^{-cz/\mu_S},\ \psi_1(z,\mu)\big|_{z\to 0}=c\,z\,,\ \psi_2(z,\mu)=\frac{e^{-cz/\mu_S}}{1/\mu_S+1/|\mu|} \quad (14)$$

For the required precision about 15-20% it is enough (see [11]) to calculate the next term in expansion given by Eq. (10). The explicit solution for the next term can be received from the following equation:

$$L_2(z,\mu,\varphi)\equiv\frac{1}{4\pi}\int_0^{2\pi}d\varphi'\int_{-1}^{1}d\mu'\,p(\cos\gamma)\int_{-\infty}^{+\infty}dz'G(z-z')\,L_1(z',\mu',\varphi')\,, \quad (15)$$

with L_1 given by Eqs. (13)-(14) and $G(z)$ by Eq. (8). The right boundary conditions has no influence on upwelling radiance distribution, so we do not discuss them here.

4. SOLUTIONS FOR THE UPWELLING RADIANCE NEAR THE SEA SURFACE

Taking integrals in Eq. (15) and summing all required terms at $z=0$ and $\mu<0$, we have the following solution for the case of homogeneous illumination:

$$L(0,\mu,\varphi)=\frac{E_w^0\,\mu_S\,b}{4\pi\,c\left(\mu_S+|\mu|\right)}\left[p(\cos\gamma_S)+\frac{b}{2\,c}\psi_p(\mu,\varphi)\right],\quad \mu<0, \quad (16)$$

here

$$\cos\gamma_S=\mu\mu_S+\sqrt{1-\mu^2}\,\sqrt{1-\mu_S^2}\,\cos\varphi, \quad (17)$$

$$\psi_p(\mu,\varphi)=|\mu|\int_0^1\frac{x_p(\mu,\mu',\varphi)}{\mu'+|\mu|}d\mu'+\frac{\mu_S}{\mu_S+|\mu|}\int_{-1}^{0}x_p(\mu,\mu',\varphi)\,d\mu', \quad (18)$$

$$x_p(\mu,\mu',\varphi)=\frac{1}{2\pi}\int_0^{2\pi}p(\cos\gamma)\,p(\cos\gamma_S')\,d\varphi',\quad \cos\gamma_S'=\mu'\mu_S+\sqrt{1-\mu'^2}\,\sqrt{1-\mu_S^2}\,\cos\varphi'. \quad (19)$$

5. HOMOGENEOUS SEA ILLUMINATED BY SUNLIGHT WITH SHADOW

Let us formulate a two-dimensional problem that takes into account inhomogeneous over horizontal axis *0x* illumination. The two-dimensional radiative transfer equation for the total angular radiance distribution is:

$$\left(\mu\frac{\partial}{\partial z}+\sqrt{1-\mu^2}\,\cos\varphi\frac{\partial}{\partial x}+c\right)L(x,z,\mu,\varphi)=\frac{b}{4\pi}\int_0^{2\pi}d\varphi'\int_{-1}^{1}p(\cos\gamma)\,L(x,z,\mu',\varphi')\,d\mu', \quad (20)$$

Let us define a one-dimensional Fourier transform by the following formulae:

$$f_k=\int_{-\infty}^{+\infty}f(x)e^{-ikx}dx\,,\quad f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}f_k\,e^{ikx}dk\,. \quad (21)$$

After taking Fourier transform Eq. (20) transfers to a one-dimensional equation for the Fourier amplitude $L_k(z,\mu,\varphi)$:

$$\left(\mu\frac{d}{dz}+\dot{c}\right)L_k(z,\mu,\varphi)=\frac{b}{4\pi}\int_0^{2\pi}d\varphi'\int_{-1}^{1}p(\cos\gamma)\,L_k(z,\mu',\varphi')\,d\mu', \qquad \begin{matrix}\dot{c}=c(1+ik\tau),\\ \tau=\sqrt{1-\mu^2}\cos\varphi/c.\end{matrix}, \quad (22)$$

Equation (22) may be obtained from the Eq.(3) by replacing the real extinction coefficient c by the complex value $\dot{c}$ given by the right side of Eqs. (22). It means that the solution of Eq. (21) can be obtained from the solution (16) by replacing all *x*-dependent values by their Fourier transforms and all instances of the extinction coefficient c by the $\dot{c}$. Taking this into account, we have the following solution to Eq. (22):

$$L_k(0,\mu,\varphi)=\frac{E_{wk}\,\mu_S\,b}{4\pi\,\dot{c}\left(\mu_S+|\mu|\right)}\left[p(\cos\gamma_S)+\frac{b}{2\dot{c}}\psi_p(\mu,\varphi)\right]. \quad (23)$$

Now we only need to calculate a Fourier transform E_{wk} of the *x*-dependent radiance distribution

$$E_{wk}=\int_{-\infty}^{+\infty}E_w(x)e^{-ikx}dx. \quad (24)$$

In our case the sun illumination incorporates a shadowing at $-w\le x<w$, where w is a half width of a shadow. The angular-space distribution of undersurface illumination is:

$$L_w^0(\mu,\varphi)=E_w(x)\delta(\mu-\mu_S)\delta(\varphi), \quad (25)$$

$$E_w(x)=E_w^0\left[1-H(w-|x|)\right]\equiv E_w^0\left\{1-0.5\left[\mathrm{sign}(w-x)+\mathrm{sign}(w+x)\right]\right\}, \quad (26)$$

here $\mathrm{sign}(x)=|x|/x$, *i.e.*, $\mathrm{sign}(x)=1,\ x>0$, $\mathrm{sign}(x)=-1,\ x<0$.

The Fourier transform of the surface illumination is:

$$E_{wk}=E_w^0\left[2\pi\,\delta(k)-\frac{2}{k}\sin(wk)\right]. \quad (27)$$

The *x*-dependent radiance distribution just below the sea surface is:

$$L(x,0,\mu,\varphi)=\frac{\mu_S\,\omega_0\,E_w^0}{4\pi\left(\mu_S+|\mu|\right)}\int_{-\infty}^{+\infty}\frac{\delta(k)-\sin(wk)/(k\pi)}{(1+ik\tau)}\left[p(\cos\gamma_S)+\frac{\omega_0}{2(1+ik\tau)}\psi_p(\mu,\varphi)\right]e^{ikx}dk. \quad (28)$$

By taking appropriate integrals we have the following solution for the undersurface upwelling radiance distribution:

$$L(x,0,\mu,\varphi)=\frac{\mu_S\,\omega_0\,E_w^0}{4\pi\left(\mu_S+|\mu|\right)}\left\{\left[1-F_1(w,\tau,x)\right]p(\cos\gamma_S)+\frac{\omega_0}{2}\left[1-F_2(w,\tau,x)\right]\psi_p(\mu,\varphi)\right\}, \quad (29)$$

$$F_1(w,\tau,x)=\frac{1}{2}\left[J_1(w+x,\tau)+J_1(w-x,\tau)+J_2(w-x,\tau)-J_2(w+x,\tau)\right], \tag{30}$$

$$\begin{aligned}F_2(w,\tau,x) &= J_3(w+x,\tau)+J_3(w-x,\tau)-\frac{1}{2}\left[J_1(w+x,\tau)+J_1(w-x,\tau)\right]\\ &+J_4(w-x,\tau)-J_4(w+x,\tau)\end{aligned}. \tag{31}$$

$$J_1(y,\tau)=\operatorname{sign}(y)\left(1-e^{-|y/\tau|}\right),\quad J_2(y,\tau)=e^{-|y/\tau|}, \tag{32}$$

$$J_3(y,\tau)=\operatorname{sign}(y)\left[1-\left(1+\frac{1}{2}\left|\frac{y}{\tau}\right|\right)e^{-|y/\tau|}\right],\quad J_4(y,\tau)=\left(1+\left|\frac{y}{\tau}\right|\right)e^{-|y/\tau|} \tag{33}$$

$$\tau=\sqrt{1-\mu^2}\cos\varphi/c \tag{34}$$

Far from the shadow in each direction (at $|w-x|,\ |w+x|>>|\tau|$) $F_1\to 0$ and $F_2\to 0$ and Eq. (29) coincides with the equation for upwelling radiance (16) for homogeneous illumination. Equations (29)-(34) include functions ψ_p and x_p, given by Eqs. (18)-(19), that involve integrations over angular variables μ and φ. In order to simplify these expressions let us substitute phase functions [12-16] inside integrals in Eqs (18)-(19) by their transport equivalents:

$$\left.\begin{aligned}p(\cos\gamma)&\to 2B+2(1-2B)\delta(\mu-\mu')\delta(\varphi-\varphi'),\\ p(\cos\gamma_S')&\to 2B+2(1-2B)\delta(\mu'-\mu_S)\delta(\varphi'-\varphi_S),\quad \varphi_S=0,\\ B&=0.5\int_{-1}^{0}p(\mu)\,d\mu.\end{aligned}\right\} \tag{35}$$

In this case after appropriate integrations we have:

$$x_p(\mu,\mu',\varphi)=2B\,\overline{p}(\mu,\mu')+2(1-2B)\,p(\cos\gamma_S), \tag{36}$$

$$\overline{p}(\mu,\mu')=\frac{1}{2\pi}\int_0^{2\pi}p(\cos\gamma)\,d\varphi', \tag{37}$$

$$\psi_p(\mu,\varphi)=4B(1-2B)|\mu|\log\frac{1+|\mu|}{|\mu|}+\frac{4\left[B(1-B)\mu_S+(1-2B)^2|\mu|\right]}{\mu_S+|\mu|}. \tag{38}$$

6. RADIANCE ABOVE THE SEA SURFACE

In order to calculate radiance distribution above the sea surface as a function of viewing angles, we have to do the following: a) we should take into account the transmission by sea-air surface from below by multiplying result by the transmission coefficient [1]:

$$T_S^{\uparrow}(\mu) = 1 - R_F^{\uparrow}(\mu) \equiv \left[1 - R_F^{\downarrow}(\mu)\right] \Big/ n_w^2, \tag{39}$$

b) we should express cosine $|\mu|$ through the zenith viewing angle θ:

$$|\mu| = \sqrt{1 - \sin^2\theta / n_w^2}, \tag{40}$$

It means that we should make the following two substitutions in Eq. (29):

$$\left.\begin{aligned} E_w^0 \to E_S^0\, T_S^{\downarrow}(\mu_S^0) T_S^{\uparrow}(\mu) = E_S \frac{\left[1 - R_F^{\downarrow}(\mu_S^0)\right]\left[1 - R_F^{\uparrow}(\mu)\right]}{\mu_S^0} \equiv \\ \equiv E_S \frac{\left[1 - R_F^{\downarrow}(\mu_S^0)\right]\left[1 - R_F^{\downarrow}(\mu)\right]}{n_w^2\, \mu_S^0}, \end{aligned}\right\} \qquad \begin{aligned} T_S^{\downarrow}(\mu_S^0) = 1 - R_F^{\downarrow}(\mu_S^0), \\ \mu_S^0 = \cos Z_S, \end{aligned} \tag{41}$$

and

$$\mu \to \sqrt{1 - \sin^2\theta / n_w^2}, \tag{42}$$

here $R_F^{\downarrow}$ is a Fresnel reflection coefficients of sunlight falling on the sea surface from above. The coefficient $R_F^{\downarrow}$ is determined by the following equation:

$$R_F^{\downarrow}(\mu) = \frac{1}{2}\left[\left(\frac{\mu - \eta}{\mu + \eta}\right)^2 + \left(\frac{n_w^2\,\mu - \eta}{n_w^2\,\mu + \eta}\right)^2\right], \qquad \eta = \sqrt{n_w^2 - 1 + \mu^2}. \tag{43}$$

7. CONCLUSIONS

This paper presents an approach to calculate radiance distribution of a sunlight reflected from a homogeneous shallow sea. The two different cases are considered: a simple homogeneous illumination by sky and sun, and an illumination with a shadow in the form of infinite stripe.

In order to efficiently calculate radiative transport in seawater the radiance of light is split into three components: unscattered, single-scattered and multiple-scattered light. Exact solutions for the first two components are found. An approximate solution to the third, multiple-scattered component, is found with the approach derived from the self-consistent theory. Because the major part of radiance distribution is calculated precisely, the resulting precision of final equations exceeds the precision of the self-consistent approach and lies in the range of 5-7%. The important feature of this approach consist of the lack of restrictions on the values of inherent optical properties. It means that this approach is valid for any type of marine and lake waters.

8. ACKNOWLEDGMENTS

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