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Analytical representation of experimental light scattering phase functions measured in seas, oceans and lake Baykal

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Abstract — This paper analyzes 20 recently released phase functions of light scattering by natural water (18 marine and 2 lake) together with 15 Petzold marine phase functions. All phase functions with regression coefficients exceeding 0.998 are represented in an analytical form that allows to compute their values at any scattering angle and the values of any derived parameter. The values of statistics with accompanied inherent optical properties that are necessary to compute modulation transfer functions are presented in a tabular form.

INTRODUCTION

In many practical cases of radiative transfer modeling it is very important to represent results of measurements of light scattering phase functions in a form of analytical equation. To obtain a meaningful analytical expression for seawater phase functions is very tricky task. Because of a high anisotropy, these phase functions and their logarithms are poorly represented by polynomials of any reasonable order. This is a consequence of the fact that Legendre polynomial decomposition of these phase functions contains several thousands of meaningful terms.

This work presents results of successful representation of new 20 phase functions measured by Mankovsky [1] and 15 phase functions published by Petzold [2] in a form of analytical expression that contains only seven parameters and represents the original numerical tables with the regression coefficients larger than 0.998 for Mankovsky phase functions and larger than 0.999 for Petzold phase functions.
Additional parameters, which are important for computations of image transfer in seawater, are also computed and presented in the form of regression that connect them with other inherent optical properties.

ANALYTICAL REPRESENTATION

Due to an extreme anisotropy of seawater scattering phase functions, numerous earlier attempts to represent marine phase functions of light scattering in analytical form either fail or produce poor results. The fist successful attempt was proposed in Ref. [3] and our analysis is an extension and improvement of the method proposed in this paper. The analysis consists of customized fitting with minimization of regression coefficient. It was found that a polynomial regression of phase function logarithm against square root of scattering angle θ produces the best results with regression coefficients distributed around value of 0.999.

$$p(\theta) = \exp\left(\sum_{n=0}^{6} c_n \left(\sqrt{\theta}\right)^n \right), \quad r^2 \sim 0.999. \tag{1}$$

The phase function in Eq. (1) is normalized according to the convenience accepted in marine optics:

$$0.5 \int_0^{\pi} p(\theta) \sin\theta \, d\theta = 1. \tag{2}$$

The seven regression coefficients $(c_0 \div c_6)$ for each of 20 Mankovsky and 15 Petzold phase functions of scattering are presented, correspondingly, in Tables 1 and 2. Two samples of comparison of experimental values of $p(\theta)$ with the values obtained with Eq. (1) are shown in Figures 1 and 2.

Equation (1) allows us to compute all important properties of waters associated with these phase functions.

Table 1. Coefficients to Eq. (1) for 20 numbered Mankovsky phase functions [1]; the numbering here starts from the top.

c_0	c_1	c_2	c_3	c_4	c_5	c_6
12.469	-45.099	62.778	-37.338	-13.128	22.983	-6.3979
10.799	-32.012	37.607	-14.917	-19.179	20.440	-5.1816
10.857	-31.494	34.853	-16.519	-9.3988	12.653	-3.3721
14.233	-66.274	112.19	-86.477	12.345	16.031	-5.6094
13.276	-55.438	90.605	-67.294	5.6820	15.804	-5.1711
8.3022	-11.706	-15.955	54.837	-67.644	37.508	-7.5588
7.9010	-8.5579	-24.545	65.429	-73.632	39.087	-7.7336
10.348	-23.204	-6.3717	70.489	-96.181	53.038	-10.455
12.094	-41.132	50.489	-24.352	-9.3636	12.934	-3.1985
14.051	-70.099	165.45	-226.06	166.75	-62.170	9.2855
10.425	-26.698	10.652	43.957	-77.833	47.219	-9.7692
11.953	-44.577	83.727	-100.57	63.020	-17.898	1.6602
11.456	-35.700	43.529	-33.044	13.188	-1.9456	-.01313
11.220	-35.314	49.886	-48.025	24.508	-4.9875	0.13011
13.446	-60.498	125.68	-154.53	103.69	-34.929	4.6379
13.254	-55.874	99.135	-97.451	47.060	-8.7793	0.10543
10.045	-21.637	-4.4404	52.645	-66.715	34.590	-6.4741
11.646	-38.767	52.491	-37.966	7.7029	4.5656	-1.7821
8.7946	-6.5846	-57.663	131.96	-124.76	55.332	-9.3715
10.742	-27.380	8.0476	41.470	-62.707	34.333	-6.5317

Table 2. Coefficients to Eq. (1) for 15 numbered Petzold phase functions [2]; the numbering here starts from the top.

c_0	c_1	c_2	c_3	c_4	c_5	c_6
12.295	-55.043	134.23	-193.23	149.02	-57.929	9.0317
11.325	-45.768	104.56	-145.18	106.71	-38.639	5.5094
11.665	-48.635	111.97	-153.75	111.37	-39.755	5.5927
12.299	-53.032	126.42	-185.22	146.91	-59.155	9.6300
12.066	-50.705	118.95	-172.19	134.04	-52.659	8.3485
12.178	-50.155	110.20	-151.59	115.70	-45.639	7.3571
12.057	-48.072	100.25	-131.05	94.482	-34.872	5.2289
11.741	-46.257	100.16	-138.28	105.08	-40.762	6.3982
9.3865	-32.619	77.634	-128.15	113.01	-48.471	8.0665
9.3403	-24.362	34.734	-40.675	25.605	-7.2741	0.74276
9.4403	-25.709	41.649	-56.117	41.540	-14.874	2.1002
11.256	-36.955	54.016	-44.470	13.073	2.3826	-1.3574
11.985	-49.121	108.22	-145.70	105.87	-38.904	5.7801
11.951	-49.667	111.72	-151.09	108.03	-37.991	5.2398
9.0729	-41.835	134.13	-243.06	221.35	-97.049	16.448

Table 3. Inherent optical properties and MTF parameters of Mankovsky phase functions counted from the top.

b,m^{-1}	c,m^{-1}	B	$\overline{\cos\theta}$	$\overline{\cos^2\theta}$	$\bar{\theta}$	$\overline{\theta^2}$
0.410	0.557	0.00453	0.98372	0.98206	0.06862	0.04359
0.299	0.403	0.01145	0.95464	0.94674	0.15650	0.11706
0.157	0.276	0.01389	0.95269	0.94809	0.15078	0.12385
0.106	0.161	0.00698	0.98196	0.98504	0.04786	0.05183
0.062	0.143	0.01936	0.94996	0.95849	0.11769	0.14375
0.518	0.626	0.01235	0.94378	0.92933	0.20512	0.14140
0.290	0.435	0.01831	0.93032	0.91973	0.22837	0.18107
0.467	0.585	0.00886	0.96009	0.95014	0.14119	0.09930
0.168	0.237	0.01228	0.96105	0.96005	0.11308	0.10426
0.115	0.168	0.01051	0.96742	0.96681	0.09095	0.08751
0.942	1.105	0.00998	0.94959	0.93454	0.17278	0.12347
0.410	-	0.01037	0.96265	0.95736	0.12691	0.09651
0.180	0.207	0.01206	0.96044	0.95740	0.12269	0.10349
0.230	0.267	0.01092	0.96092	0.95570	0.13192	0.10107
0.092	0.115	0.01524	0.95625	0.95735	0.11252	0.11809
0.115	-	0.01450	0.95861	0.96133	0.10565	0.11433
0.159	0.216	0.01634	0.94091	0.93336	0.17610	0.15199
0.117	0.157	0.02217	0.93568	0.93713	0.16646	0.17374
0.191	0.223	0.01493	0.95075	0.94716	0.15453	0.12921
0.131	0.152	0.01926	0.93812	0.93701	0.16963	0.16676

Table 4. Inherent optical properties and MTF parameters of Petzold phase functions counted from the top.

b,m^{-1}	c,m^{-1}	B	$\overline{\cos\theta}$	$\overline{\cos^2\theta}$	$\bar{\theta}$	$\overline{\theta^2}$
0.117	0.199	0.02793	0.90628	0.89909	0.24483	0.24627
0.037	0.151	0.04447	0.87091	0.87646	0.31087	0.35473
0.043	0.165	0.03872	0.88342	0.88521	0.28765	0.31757
0.275	0.470	0.01484	0.94345	0.93499	0.17361	0.14662
0.219	0.398	0.01462	0.94299	0.93351	0.17869	0.14699
1.583	1.920	0.01421	0.94170	0.92947	0.17564	0.14583
1.824	2.190	0.01428	0.94254	0.93075	0.17321	0.14361
1.205	1.330	0.01508	0.93807	0.92458	0.18946	0.15441
0.009	0.102	0.11969	0.72738	0.79676	0.53491	0.79990
0.547	0.685	0.01750	0.92816	0.91314	0.23143	0.18162
0.576	1.340	0.01643	0.93215	0.91724	0.22363	0.17075
1.284	1.480	0.01163	0.94880	0.93626	0.16532	0.12681
0.407	0.595	0.01653	0.93383	0.92111	0.19666	0.16708
0.081	0.174	0.02640	0.91466	0.91056	0.22789	0.22714
0.008	0.093	0.14792	0.66217	0.74337	0.65639	0.98568

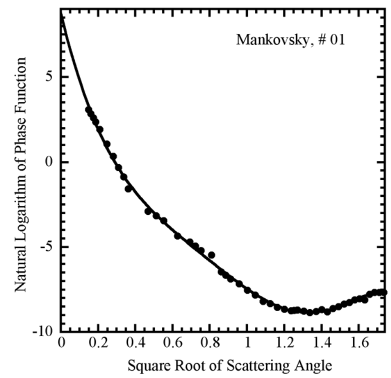

Figure 1. Mankovsky phase function # 1: experimental values and analytical representation by Eq. (1).

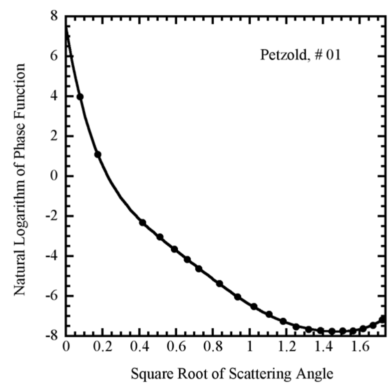

Figure 2. Petzold phase function # 1: experimental values (symbols) and analytical representation by Eq. (1).

Some of the most important optical properties of natural waters are parameters that determine image transfer and visibility in waters with given inherent optical properties (total scattering coefficient b, beam attenuation coefficient c, and probability of backscattering B, defined as a ratio of backscattering coefficient b_b to scattering coefficient b, $B = b_b / b$). The major image transfer property is a modulation transfer function (MTF) that can be estimated through average scattering angle $\bar{\theta}$, average cosine $\overline{\cos\theta}$, and average squares of these quantities ($\overline{\theta^2}$, $\overline{\cos^2\theta}$) over distribution represented by the phase function $p(\theta)$. These MTF statistics for Mankovsky and Petzold phase functions are given in Tables 3 and 4.

CONCLUSION

A new and accurate analytical representation of 20 Mankovsky and 15 Petzold phase functions of light scattering with their statistics and corresponding inherent optical properties is proposed. These functions complement previously published analytical phase function of scattering by seawater [3-5]. A FORTRAN code that computes all 35 phase functions as a function of arbitrary scattering angle is presented in an Appendix.

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APPENDIX: A FORTRAN FUNCTION TO COMPUTE MANKOVSKY AND PETZOLD PHASE FUNCTIONS OF LIGHT SCATTERING FOR ARBITRARY ANGLE

```
C ***********************************************************
      real function mplnphf(p,N,angle)
C ***********************************************************
C   Computes natural logarithm of Mankovsky or Petzold phase
C   function.  if p='m' or 'M' this function computes Mankovsky
C   phase function, if p='p' or 'P' this function computes Petzold
C   phase function. 1≤N≤15 for Petzold and 1≤N≤20 for Mankovsky.
C   angle is a scattering angle in degrees.
C ***********************************************************
      implicit     none
      integer      i, N, Nm
      real         angle, x, thet, c(0:6,35)
      character  p

      thet = angle*ATAN(1.0)/45.0
      if (p .eq. 'm' .or. p .eq. 'M') then
        Nm = N+15
        if (Nm .gt. 35) Nm = 35
      else if (p .eq. 'p' .or. p .eq. 'P') then
        Nm = N
        if (Nm .gt. 15) Nm = 15
      else
        Nm = 0
      end if
      if (Nm .ne. 0) then
       call initc(c)
       x = SQRT(thet)
        mplnphf = c(0,Nm)+x*(c(1,Nm)+x*(c(2,Nm)+x*
     &     (c(3,Nm)+x*(c(4,Nm)+x*(c(5,Nm)+x*c(6,Nm))))))
      end if

      return
      end


C ***********************************************************
      subroutine initc(c)
C ***********************************************************
      implicit  none
      integer   i, j, k
      real        c(0:6,35)

      open(11, file='c.in',status='old')
        read(11,*)
        do j = 1,15
          read(11,*) (c(i,j), i=0,6)
        end do
        read(11,*)
        do j = 1,20
          k = j+15
          read(11,*) (c(i,k), i=0,6)
        end do
      close(11)

      return
      end
C ***********************************************************
```

```
file 'c.in':
Petzold c(0:6,1:15):
 12.29463  -55.043   134.23   -193.23   149.02   -57.929   9.0317
 11.32499  -45.768   104.56   -145.18   106.71   -38.639   5.5094
 11.66488  -48.635   111.97   -153.75   111.37   -39.755   5.5927
 12.29880  -53.032   126.42   -185.22   146.91   -59.155   9.6300
 12.06609  -50.705   118.95   -172.19   134.04   -52.659   8.3485
 12.17760  -50.155   110.20   -151.59   115.70   -45.639   7.3571
 12.05656  -48.072   100.25   -131.05   94.482   -34.872   5.2289
 11.74086  -46.257   100.16   -138.28   105.08   -40.762   6.3982
  9.38649  -32.619   77.634   -128.15   113.01   -48.471   8.0665
  9.34029  -24.362   34.734   -40.67    25.605   -7.274    0.74276
  9.44031  -25.709   41.649   -56.117   41.540   -14.874   2.1002
 11.25595  -36.955   54.016   -44.47    13.073   2.3826    -1.3574
 11.98498  -49.121   108.22   -145.70   105.87   -38.904   5.7801
 11.95112  -49.667   111.72   -151.09   108.03   -37.991   5.2398
  9.07291  -41.835   134.13   -243.06   221.35   -97.049   16.4480
Mankovsky c(0:6,16:20):
 12.469   -45.099   62.778   -37.338   -13.128   22.983    -6.3979
 10.799   -32.012   37.607   -14.917   -19.179   20.440    -5.1816
 10.857   -31.494   34.853   -16.519   -9.3988   12.653    -3.3721
 14.233   -66.274   112.19   -86.477   12.345    16.031    -5.6094
 13.276   -55.438   90.605   -67.294   5.6820    15.804    -5.1711
 8.3022   -11.706   -15.955  54.837    -67.644   37.508    -7.5588
 7.9010   -8.5579   -24.545  65.429    -73.632   39.087    -7.7336
 10.348   -23.204   -6.3717  70.489    -96.181   53.038    -10.455
 12.094   -41.132   50.489   -24.352   -9.3636   12.934    -3.1985
 14.051   -70.099   165.45   -226.06   166.75    -62.170   9.2855
 10.425   -26.698   10.652   43.957    -77.833   47.219    -9.7692
 11.953   -44.577   83.727   -100.57   63.020    -17.898   1.6602
 11.456   -35.700   43.529   -33.044   13.188    -1.9456   -0.013128
 11.220   -35.314   49.886   -48.025   24.508    -4.9875   0.13011
 13.446   -60.498   125.68   -154.53   103.69    -34.929   4.6379
 13.254   -55.874   99.135   -97.451   47.060    -8.7793   0.10543
 10.045   -21.637   -4.4404  52.645    -66.715   34.590    -6.4741
 11.646   -38.767   52.491   -37.966   7.7029    4.5656    -1.7821
 8.7946   -6.5846   -57.663  131.96    -124.76   55.332    -9.3715
 10.742   -27.380   8.0476   41.470    -62.707   34.333    -6.5317
 12.558   -47.271   77.913   -82.443   48.195    -13.513   1.3736
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