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ANALYTICAL ONE-PARAMETER SEA WATER LIGHT SCATTERING PHASE FUNCTION WITH INTEGRAL PROPERTIES OF EXPERIMENTAL PHASE FUNCTIONS

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Abstract: This work proposes a new type of one-parameter two-component analytic light scattering phase function EsqaHG that has three major properties identical to the properties of experimentally measured phase functions. A single parameter that determines the shape of this new analytic seawater phase function is a probability of backscattering that is equal to the ratio of backscattering to scattering coefficients. This one-parameter analytic phase function may be a candidate for use in problems of light propagation and image transfer in seawater.

1. Introduction

Modeling of radiative transfer or visibility in sea water in many cases requires an analytical form of light scattering phase function. Existing models of scattering phase functions include Henyey-Greenstein [1], seawater two-term Henyey-Greenstein [2], and Fournier-Forand [3, 4] analytic forms of scattering phase functions. These phase functions in general could not replace experimental phase function due to the lack of certain properties. A set of experimental phase functions measured by Petzold [5] (Pacific Ocean off the California coast), Mankovsky [6-8] (Atlantic, Indian and Southern oceans, Mediterranean and Black seas, Lake Baikal), and M. Lee [9-11] (LEO-15 2000, and 2001 measurements off the New Jersey Atlantic coast) all satisfy the following properies: (1) existence of two very narrow scattering peaks - the largest one, in forward direction, and the smallest one, in backward direction; (2) strong correlation between backscattering probability and the value of phase function near the scattering angle about 140 degrees; [10, 12] (3) strong correlation between average cosine over phase function and probability of backscattering. [2, 11, 13]. Table 1 shows properties of the analytical phase functions discussed in Ref. [14] and the phase function that will be proposed in this paper. The comparison given in Tab. 1 shows that neither one of the existing analytic forms of light scattering phase functions satisfies all these three important conditions.

2. Analytic Representation of Phase Function

Let us represent a scattering phase function (PhF) in the form of a linear combination of two anisotropic phase functions, p_F with the peak forward, and p_B with the peak backward:

$$p_{VH}(\alpha,g,f,\mu) = f\ p_F(\alpha,\mu) + (1-f)\ p_B(g,\mu), \qquad (1)$$

here $\mu = \cos\theta$, θ is a scattering angle, and f, α and g are parameters. Both components of (1) and combined PhF itself are normalized according to:

$$0.5\int_{-1}^{1} p_F(\alpha,\mu)\,d\mu = 0.5\int_{-1}^{1} p_B(g,\mu)\,d\mu = 0.5\int_{-1}^{1} p_{HL}(\alpha,g,f,\mu)\,d\mu = 1. \qquad (2)$$

The backward scattering portion of the phase function (1) may be adequately represented by a Henyey-Greenstein function:

$$p_B(g,\mu) = \frac{1-g^2}{\left(1+2g\mu+g^2\right)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)(-g)^n P_n(\mu)\,, \quad 0<g<1. \qquad (3)$$

here $P_n(\mu)$ are Legendre Polynomials. The average cosine of PhF (3) is given by

$$\langle\cos\theta\rangle_B = -g, \qquad (4)$$

and backscattering probability by the formula:

$$B_B(g) = \frac{1+g}{2g}\left(1-\frac{1-g}{\sqrt{1+g^2}}\right). \qquad (5)$$

If we represent the forward scattering part of the phase function (1) by a Henyey-Greenstein term like in [2], we could not satisfy (as numerical tests show us) all three conditions outlined before. The required forward scattering term should be more anisotropic in forward direction. In order to insure higher anisotropy we choose a slower diminishing coefficient in the form of an $\exp(-\alpha'\sqrt{\theta})$, where α' is a parameter different for each phase function. That form of angular dependence at $\theta \ll 1$ is characteristic to all experimental phase functions reported in [5-10]. This gives us the following form of the forward part of the phase function:

Table 1. Properties of the phase functions discussed in [14] plus a proposed phase function.

Properties \ Phase Function	Av. Petz	OTHG	TTHG[H]	FF	EsqaHG
Number of Parameters	0 (–)	1 (+)	1 (+)	2 (+)	1 (+)
Forward Peak	Yes (+)	Yes (+)	Yes (+)	Yes (+)	Yes (+)
Shape of a Forward Peak	Good (+)	Bad (–)	Bad (–)	Good (+)	Good (+)
Backward Peak	Yes (+)	No (–)	Yes (+)	No (–)	Yes (+)
Experim. Relationship (11)	Yes (+)	No (–)	No (–)	No (–)	Yes (+)
Experim. Relationship (12)	Yes (+)	No (–)	Yes (+)	No (–)	Yes (+)
Singularity at zero angle	No (+)	No (+)	No (+)	Yes (–)	No (+)
Nice Legendre Pol. Expansion	No (–)	Yes (+)	Yes (+)	No (–)	No (–)

here: OTHG, One-Term Henyey-Greenstein [1]; TTHG[H], Two-term Henyey-Greenstein [2]; FF, Fournier-Forand [3, 4]; EsqaHG, phase function proposed here. (+) denotes a positive property, and (–) denotes a negative property.

$$p_F(\alpha,\mu) = A(\alpha)\exp\left[-\alpha\left(\frac{1-\mu}{2}\right)^{1/4}\right], \qquad \alpha > 0\,, \tag{6}$$

$$A(\alpha) = \frac{\alpha^4}{4\,\Delta(\alpha)}, \quad \Delta(\alpha) = 6(1-\exp(-\alpha)) - \alpha\exp(-\alpha)(6+3\alpha+\alpha^2) \tag{6a}$$

here α is a positive parameter. The dependence $\exp\{-\alpha[(1-\mu)/2]^{1/4}\} \approx \exp(-\alpha\sqrt{\theta/2})$ was chosen instead of $\exp(-\alpha'\sqrt{\theta})$ due to conveniences of integrability. The average cosine and backscattering probability of PhF (6) are given, respectively, by the following equations:

$$\begin{aligned}\langle\cos\theta\rangle_F = \{&6\alpha^4 + 10080[\alpha\exp(-\alpha) - (1-\exp(-\alpha))] + \alpha^2\exp(-\alpha)\times \\ &(5040 + 1680\alpha + 414\alpha^2 + 78\alpha^3 + 11\alpha^4 + \alpha^5)\}/[\alpha^4\,\Delta(\alpha)],\end{aligned} \tag{7}$$

$$\begin{aligned}B_F(\alpha) = \{&\exp(-\alpha/2^{1/4})(12 + 6\cdot 2^{3/4}\alpha + 3\sqrt{2}\,\alpha^2 + 2^{1/4}\alpha^3) \\ &-2\exp(-\alpha)(6 + 6\alpha + 3\alpha^2 + \alpha^3)\}/[2\,\Delta(\alpha)].\end{aligned} \tag{8}$$

Consequently, the final expressions for the average cosine and probability of backscattering of the phase function (1) depend on three parameters f, α, g and have the following form:

$$\langle\cos\theta\rangle(\alpha,g,f) = f\,\langle\cos\theta\rangle_F(\alpha) + (1-f)\,\langle\cos\theta\rangle_B(g) \equiv f\,\langle\cos\theta\rangle_F(\alpha) - g(1-f), \tag{9}$$

$$B(\alpha,g,f) = f\,B_F(\alpha) + (1-f)\,B_B(g). \tag{10}$$

Now we are ready to reduce a number of parameters in Eq. (1) using dependencies derived from analysis of experimental phase functions.

3. Elimination of Two Extra Parameters

Analysis of phase functions reported in Refs. [5-9] show that there are two very significant empirical relationships that are satisfied for all phase functions: 1) correlation between backscattering probability and value of the PhF at 140°:

$$B(\alpha,g,f) = \eta\, p_{VH}(\alpha,g,f,\mu_{140}), \tag{11}$$

where

$$\eta = 7.233, \quad \mu_{140} = \cos(140°) = \text{-}0.766044, \tag{11a}$$

and correlation between average cosine and back-scattering probability [8, 13]:

$$\langle\cos\theta\rangle(\alpha,g,f) = 2\frac{1-2B(\alpha,g,f)}{2+B(\alpha,g,f)}. \tag{12}$$

The first condition (11) immediately gives us an equation to express parameter f through the parameters α and g:

$$f(g,\alpha) = \frac{\Psi(g)}{\Psi(g) + \Phi(\alpha)}, \quad (13)$$

where

$$\Psi(g) = B_B(g) - \eta\, p_B(g,\mu_{140}), \quad (14)$$

and

$$\Phi(\alpha) = \eta\, p_F(\alpha,\mu_{140}) - B_F(\alpha). \quad (15)$$

The remaining two parameters α and g can be linked using relationship (12):

$$\langle\cos\theta\rangle(\alpha,g,f(\alpha))2 + B(\alpha,g,f) - 2 + 4B(\alpha,g,f(\alpha)) = 0. \quad (16)$$

It should be noted, that not every choice of forward and backward scattering components would simultaneously satisfy both restrictions (11) and (12).

4. One-Parameter Phase Function

The numerical solution of Eqs. (16), (13)-(15) and (10) gives us relationships between parameters α and g and backscattering probability B, or ratio of backscattering to scattering coefficients, $B = b_B / b$. These connections may be expressed through the following equations:

$$\alpha(B) = 7.4657\, B^{-0.25458}, \quad (17)$$

$$g(B) = 0.97847 - 0.01085\,B + 0.63542\,B^2 + 8.3409\,B^3 - 268.24\,B^4 + 2855\,B^5 - 14686\,B^6 + 30187\,B^7. \quad (18)$$

The final version of one-parameter realistic seawater phase function **EsqaHG** (**E**xponent of **sq**uare root of **a**ngle and **H**enyey-**G**reenstein) of light scattering has the following form:

$$p_{VH}(B,\mu) = \frac{\Psi[g(B)]\, p_F[\alpha(B),\mu] + \Phi[\alpha(B)]\; p_B[g(B),\mu]}{\Psi[g(B)] + \Phi[\alpha(B)]}, \quad (19)$$

with the functions Ψ and Φ expressed through (14) and (15) and α and g are given by Eqs. (17) and (18).

Expression (19) gives us a realistic representation of seawater light scattering phase function with the following properties: 1) it has two anisotropic scattering peaks, the largest one, forward, and the smallest one, backward; 2) it satisfy important relationship (11) between probability of backscattering and value of phase function at 140°; and 3) it satisfy relationship (12) between average cosine and probability of scattering $B = b_B / b$. Some realizations of these phase functions are shown in Fig. 1. It is interesting to note that all generated phase function intersect in the

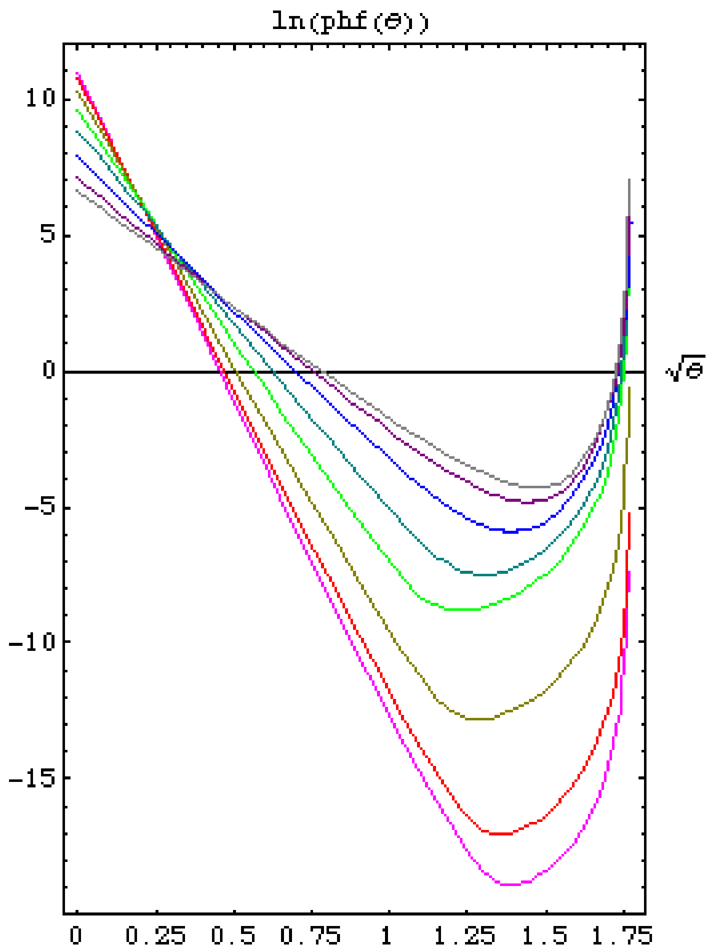

Figure 1. Plots of proposed analytical phase functions for values of $B = b_B / b$ equal to: 0.0025; 0.003; 0.005; 0.01; 0.02; 0.05; 0.1; 0.15.

vicinity of 4.5°. This feature is characteristic to all experimental phase functions [15]. Equation (19) also gives us indirect spectral dependence on a wavelength of light through the following empirical formula:

$$B(\lambda) = B(520\,\mathrm{nm})\left(\frac{520}{\lambda}\right)^{1.1}, \tag{20}$$

derived from experimental data measured by M. E. Lee and E. B. Shybanov in waters of Mobil Bay in 2003 [16].

4. Conclusions

A new type of one-parameter analytic seawater light scattering phase function EsqaHG is proposed. This phase function is taylored to satisfy three most important properties of all experimental phase functions and fairly well represents all database of existing experimental phase functions. A single parameter that determines the shape of this new analytic seawater phase function is a probability of backscattering or ratio of backscattering to scattering coefficients.

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