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One-Parameter Sea Water Light Scattering Phase Function in the Form of Legendre Polynomial Series

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***Abstract* -A new type of one-parameter seawater light scattering phase function that satisfies three most important properties of experimental phase functions is proposed.**

1. INTRODUCTION

Modeling of radiative transfer or visibility in sea water often requires an analytical form of light scattering phase function. Existing models of scattering phase functions include Henyey-Greenstein [1], seawater two-term Henyey-Greenstein [2], and Fournier-Foran [3, 4] analytic forms of scattering phase functions. These phase functions in general could not replace experimental phase function due to the lack of certain properties. A set of experimental phase functions measured by Petzold [5] (Pacific Ocean off the California coast), Mankovsky [6-8] (Atlantic, Indian and Southern Oceans, Mediterranean and Black Seas, Lake Baikal), and M. Lee [9, 10] (LEO-15 2000, and 2001 measurements off the New Jersey Atlantic coast) all satisfy the following properties: (1) existence of two very narrow scattering peaks - the largest one, in forward direction, and the smallest one, in backward direction; (2) strong correlation between backscattering probability and the value of phase function near the scattering angle about 140 degrees [11, 12]; (3) strong correlation between average cosine over phase function and probability of backscattering [2, 11, 13]. Neither one of the existing analytic forms of light scattering phase functions satisfies all these three important conditions.

2. ANALYTIC REPRESENTATION OF PHASE FUNCTION

Let us represent a scattering phase function (PhF) in the form of a linear combination of two anisotropic phase functions, p_F with the peak forward, and p_B with the peak backward:

$$p_{HL}(\varepsilon, g, f, \mu) = f\ p_F(\varepsilon,\mu) + (1-f)\ p_B(g,\mu), \qquad (1)$$

here $\mu = \cos\theta$, θ is a scattering angle, and f, ε and g are parameters. Both components of PhF (1) are normalized according to:

$$\int_{-1}^{1} p_F(\varepsilon,\mu)\, d\mu = \int_{-1}^{1} p_B(\varepsilon,\mu)\, d\mu = \int_{-1}^{1} p_{HL}(\varepsilon,\mu)\, d\mu = 2. \qquad (2)$$

The backward scattering portion of the phase function (1) may be adequately represented by a Henyey-Greenstein function:

$$p_B(g,\mu) = \frac{1-g^2}{\left(1+2g\mu+g^2\right)^{3/2}} = \sum_{n=0}^{\infty} (2n+1)(-g)^n P_n(\mu), \quad 0 < g < 1. \qquad (3)$$

here $P_n(\mu)$ are Legendre Polynomials. The average cosine of PhF (3) is given by

$$\langle \cos\theta \rangle_B = -g, \qquad (4)$$

and backscattering probability by the formula:

$$B_B(g) = \frac{1+g}{2g}\left(1 - \frac{1-g}{\sqrt{1+g^2}}\right). \qquad (5)$$

If we represent the forward scattering part of the phase function (1) by a Henyey-Greenstein term like in [2], we could not satisfy (as numerical tests show) all three conditions outlined before. The required forward scattering term should be more anisotropic in forward direction. The Henyey-Greenstein Legendre polynomial coefficient g^n decline exponentially with n: $g^n = \exp(-n \ln(1/g))$. In order to insure higher anisotropy we choose a slower diminishing coefficient in the form of a hyperbolic dependence: $(1+\varepsilon n)^{-3}$. This form of a coefficient was chosen from the following considerations: a) it should be equal to 1 at $n = 0$ (this ensures proper normalization of proposed phase function), and b) the series should converge, c) the angular behavior of this component at small angles should be consistent with the behavior of experimental phase functions [5-10]. This gives us the following form of the forward part of the phase function:

$$p_F(\varepsilon,\mu)=\sum_{n=0}^{\infty}\frac{2n+1}{(1+\varepsilon n)^3}P_n(\mu), \qquad \varepsilon>0 , \tag{6}$$

here ε is a positive parameter. The average cosine and backscattering probability of PhF (6) are given, respectively, by the following equations:

$$\langle\cos\theta\rangle_B=(1+\varepsilon)^{-3}, \tag{7}$$

$$B_F(\varepsilon)=\frac{1}{2}\left[1-\frac{3}{2(1+\varepsilon)^3}+\sum_{n=1}^{\infty}(-1)^n\frac{4n+3}{1+\varepsilon(2n+1)^3}\frac{(2n-1)!!}{(2n+2)!!}\right]. \tag{8}$$

Consequently, the final expressions for the average cosine and probability of backscattering of the phase function (1) has the following form:

$$\langle\cos\theta\rangle(\varepsilon,g,f)=f\,\langle\cos\theta\rangle_F(\varepsilon)+(1-f)\,\langle\cos\theta\rangle_B(g), \tag{9}$$

$$B(\varepsilon,g,f)=f\,B_F(\varepsilon)+(1-f)\,B_B(g). \tag{10}$$

Now we are ready to reduce a number of free parameters in Eq. (1).

3. ELIMINATION OF TWO EXTRA PARAMETERS

Analysis of experimentally measured phase functions show that there are two very significant empirical relationships that are satisfied for all phase functions: 1) correlation between backscattering probability and value of the PhF at 140°:

$$B(\varepsilon,g,f)=\eta\,p_{VH}(\varepsilon,g,f,\mu_{140}), \tag{11}$$

where

$$\eta=7.233, \quad \mu_{140}=\cos(140°)=-0.766044, \tag{11a}$$

and correlation between average cosine and back-scattering probability [8, 13]:

$$\langle\cos\theta\rangle(\varepsilon,g,f)=2\frac{1-2B(\varepsilon,g,f)}{2+B(\varepsilon,g,f)}. \tag{12}$$

The first condition (11) immediately gives us an equation to express parameter f through the parameters ε and g:

$$f(\varepsilon,g)=\frac{\Psi(g)}{\Psi(g)+\Phi(\varepsilon)}, \tag{13}$$

where

$$\Psi(g)=B_B(g)-\eta\,p_B(g,\mu_{140}), \tag{14}$$

and

$$\Phi(\varepsilon)=\eta\,p_F(\varepsilon,\mu_{140})-B_F(\varepsilon). \tag{15}$$

The remaining two parameters ε and g can be linked using relationship (12):

$$\langle\cos\theta\rangle(\varepsilon,g,f(\varepsilon,g))[1+0.5B(\varepsilon,g,f(\varepsilon,g)]+2B(\varepsilon,g,f(\varepsilon,g))=1. \tag{16}$$

It should be noted, that not every choice of forward and backward scattering components would simultaneously satisfy both restrictions (11) and (12).

4. ONE-PARAMETER PHASE FUNCTION

The numerical solution of Eqs. (16), (13)-(15) and (10) gives us relationships between parameters ε and g and backscattering probability B, or ratio of backscattering to scattering coefficients, $B=b_B/b$. These connections may be expressed through the following regressional relationships [14]:

$$\varepsilon(B)=\Omega(B), \tag{17}$$

$$g(B)=\Delta(B). \tag{18}$$

The final version of one-parameter realistic seawater phase function of light scattering has the following form,

$$p_{VL}(B,\mu)=\frac{\Psi[g(B)]}{\Psi[g(B)]+\Phi[\varepsilon(B)]}\sum_{n=0}^{\infty}\frac{2n+1}{(1+\varepsilon(B)\,n)^3}P_n(\mu)+\frac{\Phi[\varepsilon(B)]}{\Psi[g(B)]+\Phi[\varepsilon(B)]}\sum_{n=0}^{\infty}(2n+1)\,g(B)^n\,P_n(\mu), \tag{19}$$

with the functions Ψ and Φ expressed through (14) and (15) and ε and g are given by (17) and (18).

Expression (19) gives us a realistic representation of seawater light scattering phase function with the following properties: 1) it has two anisotropic scattering peaks, the largest one, forward, and the smallest one, backward; 2) it satisfy important relationship (11) between probability of back-scattering and value of phase function at 140°; and 3) it satisfy relationship (12) between average cosine and probability of scattering. Equation (19) also gives us indirect spectral dependence on a wavelength of light through the following empirical formula,

$$B(\lambda)=B(520\,\text{nm})\left(\frac{520}{\lambda}\right)^{1.1}, \tag{20}$$

derived from experimental data measured by M. E. Lee in waters of Mobil Bay in 2003.

4. CONCLUSIONS

A new type of one-parameter seawater light scattering phase function in the form of Legendre polynomial series (19) is proposed. This phase function satisfies three most important properties of all experimental phase functions and fairly well represents all database of existing experimental phase functions. A single parameter that determines the shape of this new analytic seawater phase function is a probability of backscattering or ratio of backscattering to scattering coefficients.

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[14] Functions $\Omega(B)$ and $\Delta(B)$ will be presented at the conference and will be available online.

Re: All papers by V. I. Haltrin are available online at http://haltrin.freeshell.org

* These papers contain complete tables of the measured phase functions