The Structure of Light Radiance in the Depth of the Sea for the Problems of Horizontal Visibility of Submerged Objects

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ABSTRACT

The spatial and angular structure of light radiance in the depth of seawater can be precisely computed with such numerical programs like Hydrolight. Unfortunately such calculations demand a lot of computational time if used for problems of visibility of submerged objects. In this case, due to the much greater computational efficiency and logarithmic dependence of visibility parameters on radiance values, analytical radiative transfer methods are preferable. In this presentation a new analytical radiative transfer method is proposed to solve visibility problems in seawater. The angular structure of depth-dependent light radiance in the sea is obtained in the framework of previously published self-consistent approximation. The analytical expressions proposed in this paper allow to compute a horizontal visibility of submerged objects. This approach can be applied to problems of diver visibility in shallow water bodies of arbitrary turbidity and illumination.

Keywords: shallow water, radiative transfer, ocean optics, horizontal visibility.

1. INTRODUCTION

The purpose of this paper is to obtain the simplest analytic solution to the radiative transfer problem in a shallow water with wavy surface and reflecting bottom that takes into account all orders of scattering inside water as well as all orders of scattering (or reflection) from bottom and sea surface. The approach used here is a self-consistent approach (SCA) [1-3] that was originally devised to obtain upward and downward irradiances in shallow sea with arbitrary inherent optical properties. Potentially SCA is capable to produce radiances of total light distribution in shallow sea because the main SCA radiative transfer equation has a source term that depends only on radiance of a source light and irradiances of scattered light. This potentiality was utilized in this paper to obtain total radiances of light inside shallow sea and horizontal irradiances that can be used for estimation of horizontal visibility. Calculation of horizontal visibility is considered to be the next task and can be easily performed using the results obtained in this paper.

2. RADIATIVE TRANSFER PROBLEM IN SHALLOW WATER WITH COMBINED ILLUMINATION BY SKY AND SUN

We start from the standard scalar radiative transfer equation for total radiances of light L_t in the ocean depth:

$$\left(\mu \frac{\partial}{\partial z} + c\right) L_t(z,\mu,\varphi) = \frac{b}{4\pi}\int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu'\, p(\cos\gamma)\, L_t(z,\mu',\varphi'), \qquad (1)$$

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here $c = a + b$ is an attenuation (extinction) coefficient, a is an absorption coefficient, b is a beam scattering coefficient, $p(\cos\gamma)$ is a scattering phase function of seawater normalized as $0.5\int_{-1}^{1} p(\cos\gamma)\sin\gamma\, d\gamma = 1$, γ is a scattering angle, z is a Cartesian depth coordinate originated in the water surface ($z = 0$) and going down to the bottom ($z = z_B$). the $x - y$ plane coincides with the water surface when it is calm; the zenith angle $\theta = \cos^{-1}\mu$ is measured from $0z$ axis, and the azimuth angle φ is measured from $0x$ axis. The scattering angle γ is determined by the following formula:

$$\cos\gamma = \mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\varphi - \varphi') \qquad (2)$$

here $\mu = \cos\theta$, $\mu' = \cos\theta'$. The source portion of light just below the sea surface consist of the following two components: 1) diffuse component originated from the light of the sky transformed by the wavy surface into Lambertian source, and 2) direct sunlight penetrated into the sea water.

We consider the case of shallow coastal part of the ocean with the bottom located at depth z_B and reflecting light Lambertially with bottom albedo A_B. The depth of the ocean may be less than one optical depth, $1/c$.

According to previous investigations [4] the wavy sea surface in the first approximation may be considered as a Lambertian reflector. The albedo of the surface A_S for ascending light is a function of wind speed, sea foam albedo, and refraction coefficient n_w of seawater.

We solve Eq. (1) for in-water radiances using self-consistent approach [1] to the radiative transfer of light in seawater. This approach consists of reducing the scattering phase function $p(\cos\gamma)$ to the one-parameter transport phase function, and applying two experimentally-based conditions that connect the average cosines of diffuse portion of light in water:

$$p(\cos\gamma) \to p_T(\cos\gamma) = 2B + 2(1-2B)\delta(1-\cos\gamma), \quad \delta(1-\cos\gamma) = 2\pi\,\delta(\varphi-\varphi')\delta(\mu-\mu'), \qquad (3)$$

$$\bar{\mu}_d = 1/(2-\bar{\mu}), \quad \bar{\mu}_u = 1/(2+\bar{\mu}), \qquad (4)$$

here $B = 0.5\int_{-1}^{0} p(\mu)\, d\mu$ is a backscattering probability, $\bar{\mu}_d$ is a downward average cosine, $\bar{\mu}_u$ is an upward average cosine, and $\bar{\mu}$ is a total average cosine [1-3]. The average cosines will be defined through irradiances of light later. Here and further $\delta(x)$ denotes a Dirack's delta-function.

The self-consistent approach has one huge advantage over all other approximate theories: it is valid for all possible values of inherent optical properties: extinction c, scattering b, and absorption, coefficients, *i.e.* it is good for $0 \le a,b,c < \infty$.

By applying a self-consistent condition to Eq. (1) we obtain the following equation:

$$\left(\mu\frac{\partial}{\partial z} + \alpha\right) L_t(z,\mu,\varphi) = \frac{b_B}{2\pi}\int_0^{2\pi} d\varphi' \int_{-1}^{1} d\mu'\, L_t(z,\mu',\varphi'), \qquad (5)$$

where $\alpha = a + 2b_B$ is a re-normalized extinction coefficient, and $b_B = bB$ is a backscattering coefficient.

Now, let us split the total light radiance L_t into two components: direct L_q and scattered L light, $L_t = L_q + L$. If the penetrated light of the sky and sun just below of the sea surface is expressed as:

$$L_q(0,\mu,\varphi) = L_{0D} + L_{0S}\,\delta(\mu-\mu_0)\delta(\varphi-\pi), \qquad (6)$$

where L_{0D} is a radiance of diffuse light, L_{0S} is a radiance of sunlight, then the radiance of the source will be:

$$L_q(z,\mu,\varphi)=\begin{cases}L_{0D}\,e^{-\alpha\, z/\mu}+L_{0S}\,e^{-\alpha\, z/\mu_0}\,\delta(\mu-\mu_0)\,\delta(\varphi-\pi), & \mu\ge 0,\\ 0, & \mu<0,\end{cases} \tag{7}$$

here $\mu_0=\cos\theta_0$ is a cosine of the angle θ_0 between direction of direct solar light in the water and axis $0z$. It is determined through seawater refractive index n_w and solar zenith angle $z_\otimes$ via Snellius law, $\sin z_\otimes = n_w \sin\theta_0$, or

$$\mu_0=\sqrt{1-\sin^2 z_\otimes / n_w^2}\,. \tag{8}$$

By inserting $L_t(z,\mu,\varphi)=L_q(z,\mu,\varphi)+L(z,\mu)$ into Eq. (5) with L_q given by Eq. (7), we have the following equation for scattered radiance of light inside seawater:

$$\left(\mu\frac{\partial}{\partial z}+\alpha\right)L(z,\mu)=b_B\int_{-1}^{1}d\mu'\,L(z,\mu')+Q(z), \tag{9}$$

where

$$Q(z)=b_B\left[L_{0D}\,q(\alpha\, z)+\frac{L_{0S}}{2\pi}e^{-\alpha\, z/\mu_0}\right],\qquad q(\tau)=\int_0^1 d\mu\; e^{-\tau/\mu}, \tag{10}$$

is the source function.

To proceed further from Eqs. (9)-(10) let us define diffuse irradiances and average cosines as follows:
Downward (H_d) and upward (H_u) scalar irradiances:

$$H_d(z)=\int_0^{2\pi}d\varphi\int_0^1 d\mu\, L(z,\mu)\equiv 2\pi\int_0^1 d\mu\, L(z,\mu), \tag{11}$$

$$H_u(z)=\int_0^{2\pi}d\varphi\int_{-1}^0 d\mu\, L(z,\mu)\equiv 2\pi\int_{-1}^0 d\mu\, L(z,\mu), \tag{12}$$

Downward (E_d) and upward (E_u) irradiances:

$$E_d(z)=\int_0^{2\pi}d\varphi\int_0^1 \mu\, d\mu\, L(z,\mu)\equiv 2\pi\int_0^1 \mu\, d\mu\, L(z,\mu), \tag{13}$$

$$E_u(z)=-\int_0^{2\pi}d\varphi\int_{-1}^0 \mu\, d\mu\, L(z,\mu)\equiv -2\pi\int_{-1}^0 \mu\, d\mu\, L(z,\mu) \tag{14}$$

$$\bar{\mu}_d = E_d/H_d\,,\qquad \bar{\mu}_u = E_u/H_u\,,\qquad \bar{\mu}=(E_d-E_u)/(H_d+H_d). \tag{15}$$

Using Eqs. (11)-(15) we can rewrite Eq. (9) as follows:

$$\left(\mu\frac{\partial}{\partial z}+\alpha\right)L(z,\mu)=\frac{b_B}{2\pi}[(2-\bar{\mu})E_d(z)+(2+\bar{\mu})E_u(z)]+Q(z), \tag{16}$$

where radiance of scattered light is defined through the irradiances of that light and source function of external light given by Eq. (10).

Equation (16) can be easily solved if we know analytical expressions for downward and upward irradiances and connection between average cosine $\bar{\mu}$ and inherent optical properties a and b_B. By integrating Eq. (16) over downward and upward parts of a solid angle, we obtain the following system of equations for downward and upward irradiances:

$$\hat{L}_{ik}(z)\,E_k(z) = f_i(z), \quad i = 1,2, \tag{17}$$

where index 1 corresponds to a subscript d, index 2 corresponds to a subscript u, a summation is assumed over repeated indices, $f_1(z) = f_2(z) = f(z) \equiv 2\pi\,Q(z)$, and the differential matrix operator $\hat{L}_{ik}$ is given by the following equation

$$\hat{L}_{ik}(z) = \begin{Vmatrix} \dfrac{d}{dz} + (2-\bar{\mu})(a+b_B) & -(2+\bar{\mu})b_B \\ -(2-\bar{\mu})b_B & -\dfrac{d}{dz} + (2+\bar{\mu})(a+b_B) \end{Vmatrix}, \tag{18}$$

with the average cosine $\bar{\mu}$ defined through the inherent optical properties a and b_B as follows:

$$\bar{\mu} = \sqrt{\frac{a}{a + 3b_B + \sqrt{b_B\,(4\,a + 9\,b_B)}}}\,, \text{ or } \quad \bar{\mu} = \sqrt{\frac{1-g}{1+2\,g+\sqrt{g(4+5\,g)}}}\,, \tag{19}$$

where $g = b_B/(a+b_B)$ is a Gordon's parameter.

In order to solve Eqs. (17) and, subsequently, Eq. (16) we need to formulate boundary conditions. Because we assume that both upper (sea surface) and lower (sea bottom) boundaries are Lambertian reflectors, we define the conditions as follows:

$$E_u(z_B) = A_B\,[E_d(z_B) + E_d^q(z_B)], \qquad E_d(0) = A_S\,E_u(0), \tag{20}$$

where

$$E_d^q(z_B) = \int_0^{2\pi} d\varphi \int_0^1 \mu\,d\mu\,L_q(z_B,\mu,\varphi) = 2\pi\,L_{0D}\int_0^1 e^{-\alpha\,z_B/\mu}\mu\,d\mu + L_{0S}\,\mu_0\,e^{-\alpha\,z_B/\mu_0}, \tag{21}$$

is a downward irradiance by the light of the source at the bottom. The boundary conditions (20)-(21) will be used for Eqs. (17) as well as Eq. (17).

3. SOLUTION OF RADIATIVE TRANSFER PROBLEM FOR VERTICAL IRRADIANCES OF LIGHT

Let us solve Eqs. (17) for vertical (downward and upward) irradiances of underwater diffuse light with the boundary conditions given by Eqs. (20)-(21). The complete solution to this problem is the sum of general and partial solutions to Eqs. (17). The general solution is a solution to Eqs. (17) with the right side equal to zero. The partial solution is expressed through a Green's matrix of Eqs. (17). The Green's matrix $G_{ik}(z)$ is a solution to the following matrix differential equation:

$$\hat{L}_{ik}(z)\,G_{kl}(z) = \delta(z)\,\delta_{il}, \quad i = 1,2, \tag{22}$$

where δ_{il} is a 2x2 unity matrix (or Kronecker's symbol). It is easy to show [1] that the Green's matrix is expressed as follows:

$$G_{ik}(z) = \begin{Vmatrix} 1 & R_0 \\ R_\infty & R_0R_\infty \end{Vmatrix} \frac{H(z)\,e^{-\alpha_\infty z}}{(1-R_0R_\infty)} + \begin{Vmatrix} R_0R_\infty & R_0 \\ R_\infty & 1 \end{Vmatrix} \frac{H(-z)\,e^{\alpha_0 z}}{(1-R_0R_\infty)}, \tag{23}$$

where

$$R_\infty = \left(\frac{1-\bar{\mu}}{1+\bar{\mu}}\right)^2, \qquad R_0 = \left(\frac{2+\bar{\mu}}{2-\bar{\mu}}\right) R_\infty, \tag{24}$$

α_∞ and $-\alpha_0$ are eigenvalues of matrix operator (18),

$$\alpha_\infty = \bar{\mu}\,(a+b_B) - \Delta, \qquad \alpha_0 = \bar{\mu}\,(a+b_B) + \Delta, \qquad \Delta = \sqrt{4\,a(a+2b_B) + \bar{\mu}^2\, b_B^2}\,, \tag{25}$$

and $H(x)$ is a Heavyside's or step function:

$$H(x) = \begin{cases} 1, & x \ge 0, \\ 0, & x < 0. \end{cases} \tag{26}$$

Using Green's function (23) we can write the solution to Eqs. (17) as follows:

$$E_i(z) = A\,m_i\,e^{-\alpha_\infty z} + D\,n_i\,e^{\alpha_0 z} + \int_0^{z_B} G_{ik}(z-z')\,f_k(z')\,dz', \quad i = 1,2, \quad m_i = \begin{Vmatrix} 1 \\ R_\infty \end{Vmatrix}, \quad n_i = \begin{Vmatrix} R_0 \\ 1 \end{Vmatrix}. \tag{27}$$

By inserting Eq. (23) into Eq. (27) and introducing the following three auxilary functions,

$$g(z) = 2\pi \int_0^z Q(z')\,e^{\alpha_\infty z'}\,dz', \quad h(z) = 2\pi \int_z^{z_B} Q(z')\,e^{-\alpha_0 z'}\,dz', \quad m(\tau) = 2\int_0^1 e^{-\tau/\mu}\mu\,d\mu\,, \tag{28}$$

we can simplify Eqs. (27) and obtain the following equations for downward and upward diffuse irradiances of light in shallow water body or arbitrary turbidity:

$$E_d(z) = A\,e^{-\alpha_\infty z} + DR_0\,e^{\alpha_0 z} + \frac{1+R_0}{1-R_0R_\infty}e^{-\alpha_\infty z}g(z) + R_0\,\frac{1+R_\infty}{1-R_0R_\infty}e^{\alpha_0 z}h(z), \tag{29}$$

$$E_u(z) = AR_\infty\;e^{-\alpha_\infty z} + D\,e^{\alpha_0 z} + R_\infty\,\frac{1+R_0}{1-R_0R_\infty}e^{-\alpha_\infty z}g(z) + \frac{1+R_\infty}{1-R_0R_\infty}e^{\alpha_0 z}h(z). \tag{30}$$

The two coefficients A and D in Eqs. (29)-(30) are fully determined through the two boundary conditions and can be written as follows:

$$D = \frac{A_B\,E_d^q(z_B) + (A_B - R_\infty)[\kappa\,h(0) + g(z_B)]\omega\,e^{-\alpha_\infty z_B}}{(1-R_0A_B)e^{\alpha_0 z_B} - \kappa\,(A_B - R_\infty)\,e^{-\alpha_\infty z_B}}, \qquad A = \kappa\,[\omega\,h(0) + D], \tag{31}$$

where

$$\kappa = \frac{A_S - R_0}{1 - A_S R_\infty}, \qquad \omega = \frac{1+R_0}{1-R_0\,R_\infty}, \tag{32}$$

and

$$E_d^q(z) = \pi\,L_{0D}\;m(\alpha\,z) + \mu_0\,L_{0S}\,e^{-\alpha\,z/\mu_0} \tag{33}$$

is a downward irradiance by external sources (direct and diffuse solar light and diffuse light of the sky penetrated into the water body).

4. SOLUTION OF RADIATIVE TRANSFER PROBLEM FOR RADIANCES OF SCATTERED LIGHT

Equations (9)-(10) for radiances can be rewritten as:

$$\left(\mu\frac{\partial}{\partial z}+\alpha\right)L(z,\mu)=Q_T(z), \tag{34}$$

$$Q_T(z)=b_B\left\{\frac{1}{2\pi}[(2-\overline{\mu})E_d(z)+(2+\overline{\mu})E_u(z)+L_{0S}\,e^{-\alpha\,z/\mu_0}]+L_{0D}\,q(\alpha\,z)\right\}, \tag{35}$$

where $E_d(z)$ and $E_d(z)$ are given by Eqs. (29)-(33).

With known right side the solution of Eq. (34) for scattered light radiances can be obtained as a sum of partial and general solutions:

$$L(z,\mu)=\int_0^{z_B}dz'\,Q_T(z')\,G(z-z',\mu)+\begin{cases}C_1\,e^{-\alpha\,z/\mu}, & \mu\geq 0,\\ C_2\,e^{-\alpha\,z/\mu}, & \mu<0,\end{cases} \tag{37}$$

where $G(z,\mu)$ is a Green's function of Eq. (34) that satisfies the following equation:

$$\left(\mu\frac{\partial}{\partial z}+\alpha\right)G(z,\mu)=\delta(z), \tag{38}$$

and is expressed as follows:

$$G(z,\mu)=H\left(\frac{z}{\mu}\right)\frac{e^{-\alpha\,z/\mu}}{|\,\mu\,|}, \tag{39}$$

where $H(x)$ is a Heavyside's function defined by Eq. (26). The insertion of Eq. (39) into Eq. (37) gives the following result,

$$L(z,\mu)=\begin{cases}e^{-\alpha\,z/\mu}\left[C_1+\int_0^{z}Q_T(z')e^{\alpha\,z'/\mu}\,dz'/\mu\right], & \mu\geq 0,\\ e^{-\alpha\,z/\mu}\left[C_2+\int_z^{z_B}Q_T(z')e^{-\alpha\,z'/|\mu|}\,dz'/|\,\mu\,|\right], & \mu<0.\end{cases} \tag{40}$$

The constants C_1 and C_2 are determined by the boundary conditions given by Eqs. (20)-(21),

$$C_1=A_S\,(C_2+\varepsilon_0),\quad C_2=A_B\,\frac{\varepsilon_B+A_S\,\varepsilon_0+L_{0D}+\mu_0\,(L_{0S}/\pi)\,e^{-\alpha\,z_B/\mu_0}}{m_+\,(1-A_B\,A_S\,\rho)}, \tag{41}$$

where

$$\varepsilon_0=2\int_0^{z_B}Q_T(z)\,n(\alpha\,z)\,dz,\qquad \varepsilon_B=2\int_0^{z_B}Q_T(z)\,n[\alpha(z_B-z)]\,dz, \tag{42}$$

$$\rho = \frac{m_-}{m_+} \equiv \int_0^1 \mu\, d\mu\, e^{-\alpha z_B/\mu} \Big/ \int_0^1 \mu\, d\mu\, e^{\alpha z_B/\mu} \leq 1, \tag{43}$$

$$m_- = m(\alpha z_B) \equiv 2\int_0^1 \mu\, d\mu\, e^{-\alpha z_B/\mu} > 0, \quad m_+ = m(-\alpha z_B) \equiv 2\int_0^1 \mu\, d\mu\, e^{\alpha z_B/\mu} > 0, \quad n(z) = \int_0^1 e^{-\alpha z/\mu}\, d\mu\,. \tag{44}$$

Equations (40)-(44) fully define scattered light radiances. By adding direct radiance (7) we obtain the total radiances in the shallow sea:

$$L_t(z,\mu,\varphi) = \begin{cases} e^{-\alpha z/\mu}\left[L_{0D} + C_1 + \int_0^z Q_T(z')e^{\alpha z'/\mu}\, dz'/\mu\right] + L_{0S}\, e^{-\alpha z/\mu_0}\,\delta(\mu-\mu_0)\delta(\varphi-\pi), & \mu \geq 0, \\ e^{-\alpha z/\mu}\left[C_2 + \int_z^{z_B} Q_T(z')e^{-\alpha z'/|\mu|}\, dz'/|\mu|\right], & \mu < 0. \end{cases} \tag{45}$$

Equation (45) represents a self-consistent solution to the radiative transfer problem for radiance of light in shallow water with wavy Lambertian surface and Lambertially reflecting bottom. It takes into account multiple scattering inside water column and from the bottom and the surface, and it is valid in the whole range of variability of inherent optical properties ($0 \leq a \leq \infty,\ 0 \leq b \leq \infty,\ 0 \leq B \leq 0.5$) and, consequently, can be used for waters with arbitrary turbidity and absorption. For modeling purposes inherent optical properties can be generated, for example, using approach proposed in Ref. [5].

5. HORIZONTAL IRRADIANCES OF LIGHT IN SHALLOW WATERS

The horizontal irradiance in shallow water is a function of depth and azimuth angle φ between the direction of solar rays and the direction of light we measure. It is given by the following equation:

$$E_h(z,\varphi) = \pi\left\{[L_{0D} + C_1]\Phi(\alpha z) + C_2\,\Phi(-\alpha z) + \int_0^{z_B} \Psi(\alpha[z-z'])Q_T(z')\, dz'\right\} + \begin{cases} L_{0S}\sqrt{1-\mu_0^2}\, e^{-\alpha z/\mu_0}\cos(\pi-\varphi), & |\pi-\varphi| < \pi/2, \\ 0, & |\pi-\varphi| > \pi/2. \end{cases} \tag{46}$$

here

$$\Phi(\tau) = \int_0^1 \sqrt{1-\mu^2}e^{-\tau/\mu}d\mu, \qquad \Psi(\tau) = \int_0^1 \sqrt{1-\mu^2}e^{-|\tau|/\mu}\, d\mu/\mu. \tag{47}$$

the function $\Psi(\tau)$ has an infinite, narrow and integrable peak at $\tau = 0$, so it can be replaced by a delta function:

$$\Psi(\tau) \cong \frac{\pi}{4}\delta(\tau), \quad \delta(\alpha z) = \frac{1}{\alpha}\delta(z). \tag{48}$$

This significantly simplifies the expression for horizontal irradiance:

$$E_h(z,\varphi) = \pi\left\{[L_{0D} + C_1]\Phi(\alpha z) + C_2\,\Phi(-\alpha z) + \frac{\pi}{4\alpha}Q_T(z)\right\} + \begin{cases} L_{0S}\sqrt{1-\mu_0^2}\, e^{-\alpha z/\mu_0}\cos(\pi-\varphi), & |\pi-\varphi| \leq \pi/2, \\ 0, & |\pi-\varphi| > \pi/2, \end{cases} \tag{49}$$

with the coefficients $C_1$and C_2 given by Eqs. (41). It is necessary to note that the third component in the right part of Eq. (49) is not equal to zero only on a sunny side of the viewing direction $|\pi - \varphi| \le \pi / 2$.

6. CONCLUSION

We obtained here a simple self-consistent solution, given by Eq. (45), to the radiative transfer problem in a shallow sea with wavy surface and diffusely reflecting bottom. This solution takes into account multiple scattering of light inside water body as well as multiple reflections from bottom and wavy surface. It is valid for waters with arbitrary absorption and turbidity. The obtained solution was used to calculate horizontal azimuthally dependent irradiances in the shallow water with reflecting bottom and wavy surface. Expression (49) for horizontal light irradiances is a principal equation that will allow to solve the problem of horizontal (or diver) visibility of an object submerged in a shallow water with arbitrary inherent optical properties.

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8. REFERENCES †

1. V. I. Haltrin, "Self-consistent approach to the solution of the light transfer problem for irradiances in marine waters with arbitrary turbidity, depth and surface illumination," *Appl. Optics*, **37**, 3773-3784 (1998).

2. V. I. Haltrin, "Apparent optical properties of the sea illuminated by Sun and sky: case of the optically deep sea," *Appl. Optics*, **37**, 8336-8340, (1998).

3. V. I. Haltrin, "Diffuse reflection coefficient of a stratified sea," *Applied Optics*, **38**, 932-936 (1999).

4. V. I. Haltrin, W. E. McBride III, and R. A. Arnone, "Spectral Approach to Calculate Specular Reflection of Light from Wavy Water Surface," – pp. 133-138 in *Proceedings of D. S. Rozhdestvensky Optical Society: International Conference Current Problems in Optics of Natural Waters* (ONW'2001), St. Petersburg, Russia, 2001.

5. V. I. Haltrin, "Chlorophyll based model of seawater optical properties, "Applied *Optics*, **38**, 6826-6832, (1999).

† The referenced articles can be viewed and downloaded in a PDF format from the following web site: <http://haltrin.freeshell.org>.