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A Method and Algorithm of Computing Apparent Optical Properties of Coastal Sea Waters

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Abstract – A new approach is proposed for the calculation of irradiances, diffuse attenuation coefficients and diffuse reflectances in waters with arbitrary scattering and absorption coefficients, arbitrary conditions of illumination and a bottom with Lambertian albedo. The two-stream approach adopted here utilizes experimental dependencies of mean cosines from inherent optical properties in order to achieve appropriate accuracy. This approach can be successfully used for calculation of apparent optical properties in both open and coastal oceanic waters, lakes and rivers.

INTRODUCTION

For many practical applications of remote sensing such as the inference of the diffuse attenuation coefficient and component inversion it is sufficient to know only the integral characteristics of the light field such as upward and downward irradiances or reflectances. Present models used in remote sensing applications for radiative transfer employ simple two-flow or quasi-single scattering approximations which suffer from limited validity over the dynamic range of optical properties found in the ocean. However the limitation to open ocean water types restricts the general usage of these models. Remote sensing applications would be greatly enhanced if we add to it a simple model that can be used over all water types, turbid to open ocean. We present a semi-empirical model that incorporates laboratory and *in situ* measurements of optical properties [1, 2] to encompass the entire range of natural waters.

We start from an exact equation for irradiances derived from the scalar transfer equation. To make this equation solvable it is necessary, however, to approximate the resulting coefficients of the system of two-flow equations. Due to the inaccuracy inherent in the approximations, previous approaches [3] have resulted in insufficient accuracy over some portions of the natural range of optical parameters. We use two main steps to reduce the exact, but analytically unsolvable, system of equations to an approximate system which can be easily solved. The first step consists of replacing the initial arbitrary phase function with the transport phase function. This greatly simplifies the equations, but introduces excessive error. We reclaim the lost accuracy, in the next step, by introducing empirical relationships between the upward and downward cosines and total mean cosine derived from laboratory and *in situ* data [1].

In the case of coastal waters it will be more consistent to take into account variability of upward, downward and total mean cosines ($\bar{\mu}_u,\ \bar{\mu}_d,\ \bar{\mu}$) or their functional dependence on inherent optical properties. The experimental (modeled and measured *in-situ*) data of Timofeyeva [1, 2] show that with the change of $x = B\omega_0 / (1-\omega_0 + B\omega_0)$ between 0 and 1 (here ω_0 is the single-scattering albedo and B is the probability of backscattering) the total mean cosine $\bar{\mu}$ also varies between 0 and 1, the upward mean cosine $\bar{\mu}_u$ decreases from 1 to ~0.25 at $x \sim 0.08$ and then increases to 0.5 at $x = 1$, and the downward mean cosine $\bar{\mu}_d$ decreases from 1 to 0.5.

The main purpose of this work is to obtain equations which relate inherent optical to apparent optical properties for any input radiance distribution. These equations, which are convenient and precise, are valid in the complete range of variability of optical properties of natural water.

In transfer theory, requirements of both simplicity and precision are mutually exclusive. For a successful resolution of the problem, therefore, we have accepted a compromise by determining the degree of simplicity and precision.

In solving our problem we will use the *self-consistent method* proposed in [4]. For a better understanding of the idea of this method, we quote an example from classical mechanics [5], from which it was adopted. Suppose we have to obtain the equation of motion of a material body around some center of attraction. The law of attraction is unknown to us, or it is known only partially, but in addition we have some information on the shape of trajectories in the form of dependencies between integral parameters of these trajectories. This problem can be solved provided we use the available information to constrain the acceptable solutions. In this example the knowledge of additional information on *consequences* (trajectory parameters) has made it possible to compensate for the lack of information on *causes* (attraction forces).

In the theory of radiative transfer the main *causes* are the inherent optical properties such as the scattering law characteristics (volume scattering and single scattering albedo), and the main *consequences* are the apparent optical properties, such as the angular distribution of radiance, as a functions of depth. In general, the volume scattering function is only approximately known, with unknown precision. It is impossible in general to calculate the volume scattering function of an actual medium because in many cases the shape of the scattering particles is irregular and often *exotic*, with the optical characteristics of these particles known only approximately. Experimental measurements of the volume scattering function in the small-angles regime becomes complicated due to difficulty in discriminating between unscattered and forward scattered light. The measurements of the volume scattering function in the range of angles close to the backward direction are in principal impossible because one cannot install a receiver before or behind an emitter without considerable distortion in the process of measurement. To overcome this, beam splitting of backscattered light has been utilized with some success. On the contrary because, as a rule, the angular distribution of the scattered light at depth is always less anisotropic than the volume scattering function, and the anisotropy of the direct light of the outer sources is known, the measurements of radiance distribution are less difficult, and the precision of these measurements is restricted only by the perfection of the measuring device.

Thus, in our attempts to solve the problem of light field calculation in a scattering and absorbing medium, we restrict ourselves to the simplest transport approximation of the volume scattering function. The information, which we lose through this simplification, is restored by accepting functional dependencies between integral parameters of the radiance angular distribution, which are derived from an approximation of experimental data.

FORMULATION OF THE PROBLEM

We shall start from the scalar equation describing the transport of optical radiation in a layer of a scattering and absorbing medium of thickness H

$$\left(\cos\theta\frac{\partial}{\partial z}+c\right)L_t(z,\theta,\varphi)=\frac{b}{4\pi}\int L_t(z,\theta',\varphi')\,p(\gamma)\,d\Omega', \quad (1)$$

where $L_t(z,\theta,\varphi)$ is the spectral density of the energetic radiance (or, simply, radiance) of light, θ and φ are the zenith and azimuth angles in the direction of light propagation, measured from the positive direction of the $0z$-axis, $c=a+b$ is the extinction (attenuation) coefficient, a is the absorption coefficient, b is the scattering coefficient, $d\Omega\equiv\sin\theta\,d\theta\,d\varphi$ is the element of solid angle, $p(\gamma)$ is the volume scattering function. Here γ is the light scattering angle, which is determined from the relation: $\cos\gamma=\mu\mu'+\sqrt{(1-\mu^2)(1-\mu'^2)}\cos(\varphi-\varphi')$, where $\mu=\cos\theta$, $\mu'=\cos\theta'$, and the phase function is normalized as follows: $\int p(\gamma)\,d\Omega'=4\pi$. The system of coordinates here is chosen so that the xy-plane coincides with the outer boundary of the medium on which the radiation is incident, while the $0z$-axis is oriented into the medium.

In an anisotropic light-scattering media the phase function $p(\gamma)$ has a distinct diffraction peak near $\gamma=0$. The light rays scattered in a small solid angle near the forward direction $(\gamma\cong 0)$ form the halo part of the scattered light and are, for many applications, indistinguishable from the unscattered rays. This suggests that the halo part of the rays should not be regarded as scattered rays, *i.e.* the forward diffraction peak can be eliminated from the volume scattering function [4].

We separate the main part of the halo rays by representing the volume scattering function as a sum of isotropic and anisotropic components:

$$p(\gamma)=2B+(1-2B)\,p_h(\gamma),$$
$$p_h(\gamma)=\left[p(\gamma)-2B\right]/(1-2B),\quad \int p_h(\gamma)\,d\Omega'=4\pi, \quad (2)$$

where $B=0.5\int_{\pi/2}^{\pi}p(\gamma)\sin\gamma\,d\gamma$ is the probability of scattering into the backward hemisphere. When the elongation of the phase function is increased, the relation $\lim_{B\to 0}p_h(\gamma)=2\delta(1-\cos\gamma)\equiv 4\pi\,\delta(\varphi-\varphi')\,\delta(\mu-\mu')$ exists, where $\delta(x)$ is the Dirac delta-function. As $B\to 0$ the phase function $p(\gamma)$ equals the transport phase function

$$p_t(\gamma)=2B+2(1-2B)\,\delta(1-\cos\gamma),\quad \int p_t(\gamma)\,d\Omega'=4\pi. \quad (3)$$

Substituting $p(\gamma)\equiv p_t(\gamma)+(1-2B)[p_h(\gamma)-2\delta(1-\cos\gamma)]$ into (1), we get

$$\left(\mu\frac{\partial}{\partial\tau}+\alpha\right)L_t(\tau,\theta,\varphi)=\frac{x}{2\pi}\int L_t(\tau,\theta',\varphi')\,d\Omega'+$$
$$\frac{x(1-2B)}{4\pi B}\int[p_h(\gamma)-2\delta(1-\cos\gamma)]\;L_t(\tau,\theta',\varphi')\,d\Omega', \quad (4)$$

where $\alpha=1+x$, $x=b_b/(a+b_b)\equiv B\omega_0/(1-\omega_0+B\omega_0)$, $b_b=bB$ is the backscattering coefficient; $\omega_0=b/(a+b)$ is the single scattering albedo, and $\tau=z(a+b_b)$ is the *transport* optical depth.

Let $L_q(\mu,\varphi)$ be the radiance of external sources at $\tau=+0$ $(z=+0)$ (below water surface) and $L(\tau,\mu,\varphi)$ be the radiance of the scattered component minus the halo rays at the optical depth τ. In this case the total radiance distribution $L_t(\tau,\mu,\varphi)\equiv L_t(z,\mu,\varphi)$ can be expressed as

$$L_t(\tau,\mu,\varphi)=L(\tau,\mu,\varphi)+L_q(\mu,\varphi)\,\theta(\mu)\exp(-\alpha\tau/\mu), \quad (5)$$

where $\theta(\mu)$ is the Heavyside (or step) function defined by: $\theta(\mu)=1,\mu>0;\theta(\mu)=0,\mu\le 0$. In this case α is the attenuation coefficient for the sum of forward and halo rays. In (5) we assume that either the layer of scattering medium is optically thick $\{\alpha(a+b_b)H\equiv(a+2b_b)H>>1\}$, or that its lower boundary reflects light according to Lambert's law. Substituting (5) into (4), we obtain an equation for the radiance of the scattered light (without halo)

$$\left(\mu\frac{\partial}{\partial\tau}+\alpha\right)L(\tau,\mu,\varphi)=\frac{x\,E_0(\tau)+g(\tau,\mu,\varphi)+\Delta(\tau,\mu,\varphi)}{2\pi}, \quad (6)$$

where $E_0(\tau)$ is the scalar irradiance by diffuse light,

$$E_0(\tau)=\int_0^{2\pi}d\varphi\int_{-1}^{1}L(\tau,\mu,\varphi)\,d\mu, \quad (7)$$

$g(\tau,\mu,\varphi)$ is the source function

$$g(\tau,\mu,\varphi)=\frac{x}{2B}\int_0^{2\pi}d\varphi\int_0^1 p(\gamma)\,L_q(\mu',\varphi')\,e^{-\frac{\alpha\tau}{\mu'}}d\mu' \\ -\left[2\pi\,x\,(1-2B)/B\right]L_q(\mu,\varphi)\,e^{-\frac{\alpha\tau}{\mu}}, \quad (8)$$

$$\Delta(\tau,\mu,\varphi)=\left[2xB/(1-2B)\right]\int_0^{2\pi}d\varphi'\times \\ \int_{-1}^1\left[p_h(\gamma)-2\,\delta(1-\cos\gamma)\right]L(\tau,\mu',\varphi')\,d\mu'. \quad (9)$$

Equation (6) is totally equivalent to (1). Introduction of the function $\Delta(\tau,\mu,\varphi)$ in (6) corresponds to including the halo rays in the nonscattered light. The expression (9) completely vanishes in two limiting cases: (a) for isotropic scattering: $p(\gamma)=1$ at $B=0.5$, and (b) for extremely anisotropic scattering: $p(\gamma)=2\,\delta(1-\cos\gamma)$ at $B=0$.

ENHANCED TWO-FLOW APPROXIMATION

Equation (6) for arbitrary $p(\gamma)$ cannot be solved analytically. But if we neglect the term Δ compared to $x\,E_0+g$, we reduce the problem to the case of an exactly solvable transport approximation. However in doing so we decrease the accuracy of our results.

In order to overcome this shortcoming we propose to solve (6) in the terms of self-consistent approximation [4]. This consists of neglecting the value Δ in comparison with $x\,E_0+g$, and but then taking it into account later by evaluating the quantity $x\,E_0+g$ through the two-flow approximation. This technique requires empirical dependencies between the integral parameters of radiance distribution, which have been derived from experimental data.

We introduce downward (E_d) and upward (E_u) irradiances by diffuse light (without halo)

$$E_d(\tau)=\int_0^{2\pi}d\varphi\int_0^1 L(\tau,\mu,\varphi)\,\mu\,d\mu, \quad (10)$$

$$E_u(\tau)=-\int_0^{2\pi}d\varphi\int_{-1}^0 L(\tau,\mu,\varphi)\,\mu\,d\mu, \quad (11)$$

and scalar irradiances by diffuse light (without halo)

$$E_{0d}(\tau)=\int_0^{2\pi}d\varphi\int_{-1}^0 L(\tau,\mu,\varphi)\;d\mu, \quad (12)$$

$$E_{0u}(\tau)=\int_0^{2\pi}d\varphi\int_{-1}^0 L(\tau,\mu,\varphi)\;d\mu. \quad (13)$$

Average downward and upward cosines of the diffuse light distribution (without halo) would be

$$\mu_d(\tau)=E_d(\tau)/E_{0d}(\tau),\quad \mu_u(\tau)=E_u(\tau)/E_{0u}(\tau). \quad (14)$$

We introduce average cosine for the diffuse light distribution

$$\mu(\tau)=\int_0^{2\pi}d\varphi\int_{-1}^1 L(\tau,\mu,\varphi)\,\mu\,d\mu\Big/\int_0^{2\pi}d\varphi\int_{-1}^1 L(\tau,\mu,\varphi)\;d\mu, \quad (13)$$

Applying the operators $\int_0^{2\pi}d\varphi\int_0^1 d\mu\ldots$, $\int_0^{2\pi}d\varphi\int_{-1}^0 d\mu\ldots$ on (6) with $\Delta=0$, using (10)-(14) and replacing the average cosines $\mu_i(\tau)$ $(i=1,2)$ (indices 1 and 2 are equivalent, respectively, to the indices d and u) by their values deep in the layer, we obtain from (6)

$$D_{ik}(\tau)\,E_k(\tau)=f_i(\tau),\qquad i,\,k=1,\,2, \quad (16)$$

where

$$D_{ik}(\tau)=\begin{pmatrix}\dfrac{\partial}{\partial\tau}+\dfrac{1}{\mu_d} & -\dfrac{x}{\mu_u}\\ -\dfrac{x}{\mu_d} & -\dfrac{\partial}{\partial\tau}+\dfrac{1}{\mu_u}\end{pmatrix}, \quad (17)$$

$$f_1(\tau)=(x/B)\int_0^{2\pi}d\varphi\int_0^1[2B-\psi(\mu)]\,L_q(\mu,\varphi)\,e^{-\frac{\alpha\tau}{\mu}}d\mu\ , \quad (18)$$

$$f_2(\tau)=(x/B)\int_0^{2\pi}d\varphi\int_0^1\psi(\mu)\,L_q(\mu,\varphi)\,e^{-\frac{\alpha\tau}{\mu}}d\mu\ , \quad (19)$$

$$\psi(\mu)=\frac{1}{2}\int_0^1 p_m(-\mu',\mu)\,d\mu',\quad p_m(\mu',\mu)\equiv\frac{1}{2\pi}\int_0^{2\pi}p(\gamma)\,d\varphi. \quad (20)$$

Here and further in this paper repeated indices imply summation.

We shall look for solution of (16) in the form of a sum of the general and particular solutions

$$E_i(\tau)=A\,a_i\exp(\varepsilon_1\tau)+P\,p_i\exp(\varepsilon_2\tau) \\ +\int_0^{\tau_H}G_{ik}(\tau-\tau')\,f_k(\tau')\,d\tau',\quad i,\,k=1,\,2, \quad (21)$$

where ε_1 and ε_2 are eigenvalues of equation (16), and G_{ik} is the Green's matrix of this equation, which satisfies the equation $D_{il}\,G_{lk}(\tau)=\delta_{ik}\,\delta(\tau)$.

Before writing down the solutions of (21) let us find eigenvalues ε_1 and ε_2. Inserting (21) into (16) we obtain

$$\varepsilon^2-\varepsilon\left(1/\mu_u-1/\mu_d\right)-\left(1-x^2\right)\big/\left(\mu_u\,\mu_d\right)=0, \quad (22)$$

Before solving the quadratic equation (22), let us determine the functional dependencies of the average cosines μ_u, μ_d on optical parameters of the medium. In the majority of two-stream theories [3, 7] the quantities μ_u and μ_d are considered as independent of the optical properties of the medium and such an assumption leads to significant reduction of accuracy in those approaches.

In this work we shall adopt the following two suppositions of the self-consistent approach:

(a) We shall assume that neglecting Δ in (18)-(19) does not have any influence on the magnitude of the negative eigenvalue of the system of equations (16), *i.e.* we assume that it is equal to its exact value ε_e

$$-\varepsilon_1=\varepsilon_e\equiv(1-x)/\bar{\mu}\ . \quad (23)$$

(b) We will suppose that values μ_u and μ_d are functions of the mean cosine $\bar{\mu}$ and adopt the functional dependencies $\mu_u(\bar{\mu})$ and $\mu_d(\bar{\mu})$ which results from experiment.

After accepting these assumptions we can express the parameter x on $\bar{\mu}$ by installing (23) into (22)

$$x=[\bar{\mu}+\mu_u(\bar{\mu})][\mu_d(\bar{\mu})-\bar{\mu}]\big/\left[\mu_u(\bar{\mu})\,\mu_d(\bar{\mu})+\bar{\mu}^2\right], \quad (24)$$

By inserting into (24) experimental values of $\bar{\mu}$, μ_u and μ_d [1, 2], we obtain the corresponding values of x (see Table1). Table 1 makes it possible to obtain empirical dependencies which connect mean cosines, diffuse reflectance R_∞ and coefficient $k=R_\infty(x)/x$ to the medium parameter $x=b_b/(a+b_b)\equiv B\omega_0\big/\left(1-\omega_0+B\omega_0\right)$.

By imposing the condition of realization of asymptotic behavior of R_∞ at small x and $(1-x)$ given in [8]:

Table 1.

$\bar\mu$	$\bar\mu_d$	$\bar\mu_u$	R_∞	x	R_∞/x
0	0.5	0.5	1.0	1.0	1.0
0.1	0.5249	0.4831	0.671	0.9408	0.7132
0.2	0.5525	0.4545	0.443	0.7970	0.5550
0.3	0.5834	0.4202	0.283	0.6179	0.4580
0.4	0.6184	0.3745	0.171	0.4439	0.3852
0.5	0.6566	0.3311	0.095	0.2959	0.3211
0.6	0.7008	0.3003	0.048	0.1802	0.2664
0.7	0.7536	0.2857	0.0207	0.0967	0.2141
0.8	0.8217	0.3610	0.0082	0.0413	0.1985
0.9	0.9033	0.6849	0.0016	0.0101	0.1584
1.0	1.0	1.0	0.0	0.0	0.25
Experiment [1, 2]				Eqn. (24)	

$$R_\infty(x)=\begin{cases} x/4, & x<<1 \quad (\text{or } 1-\bar\mu<<1),\\ 1-4\sqrt{(1-x)/6}, & 1-x<<1 \quad (\text{or } \bar\mu<<1),\end{cases} \quad (25)$$

we can obtain from the data of Table 1 the following equations

$$\bar\mu = a_0+(1-a_0)\sqrt{1-x}+\sum_{n=1}^{6} a_n x^{\frac{n}{3}}, \quad (26)$$

$$\mu_d = \left[1-\bar\mu(1-\bar\mu)^2\sum_{n=0}^{3} b_n\bar\mu^{2n}\right]\Big/(2-\bar\mu), \quad (27)$$

$$\mu_u = \left[1-\bar\mu(1-\bar\mu)^2\exp\left(\sum_{n=0}^{4} c_n\bar\mu^{2n}\right)\right]\Big/(2-\bar\mu), \quad (28)$$

$$R_\infty \equiv \frac{1-\bar\mu/\mu_d}{1+\bar\mu/\mu_u} = \frac{(1-\bar\mu)^2\left[1-\bar\mu(1-\bar\mu)^2\sum_{n=0}^{3} b_n\bar\mu^{2n}\right]}{2-(1-\bar\mu)^2\left[1+\bar\mu\exp\left(\sum_{n=0}^{4} c_n\bar\mu^{2n}\right)\right]}, \quad (29)$$

$$k=\frac{1}{4}+d_0\left(1-\sqrt{1-x}\right)+d_1\sqrt[3]{x}+\sqrt[3]{x^2}(1-x)\sum_{n=2}^{6} d_n x^{n-2}. \quad (30)$$

The values of coefficients a_n, b_n, c_n and d_n are given in Table 2. The correlation coefficients between the quantities of parameters given in Table 1 and those computed by (26)-(30) in all cases exceed 0.99, and mean quadratic deviations are less then 3%.

The functional dependencies of the experimental values of $\bar\mu, \mu_u, \mu_d, R_\infty$ and values computed with (26)-(30) are shown on Fig.1.

With inclusion of empirical equations (26)-(28), (22) and (23) will give us the following dependencies for the eigenvalues ε_1 and ε_2:

Table 2.

n	a_n	b_n	c_n	d_n	r_n
0	0.5918	0.0326	-0.0131	1.6330	0.7500
1	-0.7937	0.1661	8.4423	-0.8830	0.3750
2	4.8350	0.7785	-15.6605	0.4631	25.3315
3	-22.8150	0.0228	21.8820	2.3442	-83.6066
4	42.6859		-11.2257	-6.0841	24.7228
5	-35.8945			7.5933	130.6733
6	11.3905			-3.8215	-105.6769

$$-\varepsilon_1 \equiv \alpha_\infty = (1-x)/\bar\mu, \quad (31)$$

$$\varepsilon_2 \equiv \alpha_0 = 1/\mu_u - 1/\mu_d + (1-x)/\bar\mu, \quad (32)$$

It is easy to show that the Green's matrix $G_{ik}(\tau)$ of (16) has the following form: (33)

$$G_{ik}(\tau)=\begin{pmatrix} 1 & R_0 \\ R_\infty & R_0 R_\infty \end{pmatrix}\frac{\theta(\tau)e^{-\alpha_\infty\tau}}{1-R_0 R_\infty}+\begin{pmatrix} R_0 R_\infty & R_0 \\ R_\infty & 1 \end{pmatrix}\frac{\theta(-\tau)e^{\alpha_0\tau}}{1-R_0 R_\infty},$$

and $\quad a_1=p_2=1, \quad a_2=R_\infty, \quad p_1=R_0,$ (34)

where
$$R_\infty = \lim_{\tau\to\infty} E_u(\tau)/E_d(\tau) \equiv \left(1-\bar\mu/\mu_d\right)/\left(1+\bar\mu/\mu_u\right) = \mu_u(1-\alpha_\infty\mu_d)/(x\,\mu_d) = x\,\mu_u/\left[\mu_d(1+\alpha_\infty\mu_u)\right], \quad (35)$$

is the diffuse reflectance of an infinitely thick layer at $\tau>>1$,

$$R_0 = \lim_{\tau\to\infty} E_{0u}(\tau)/E_{0d}(\tau) \equiv \left(\mu_d-\bar\mu\right)/\left(\mu_u+\bar\mu\right) = R_\infty\mu_d/\mu_u = \left(1-\alpha_\infty\mu_d\right)/x = x/\left(1+\alpha_\infty\mu_u\right), \quad (36)$$

Substituting (33) and (34) into (21) and imposing boundary conditions at $\tau=0$ and $\tau_H \equiv H(a+b_b)$:

$$E_d(0)=E_0, \quad E_u(\tau_H)=A_B[E_d(\tau_H)+E_d^f(\tau_H)] \quad (37)$$

where A_B is the albedo of the lower boundary and $E_d^f(\tau)$ is the total flux of the unscattered and halo rays

$$E_d^f(\tau)=\int_0^{2\pi}d\varphi\int_0^1 L_q(\mu,\varphi)\exp(-\alpha\tau/\mu)\,\mu\,d\mu, \quad (38)$$

we obtain the equations for the descending and ascending

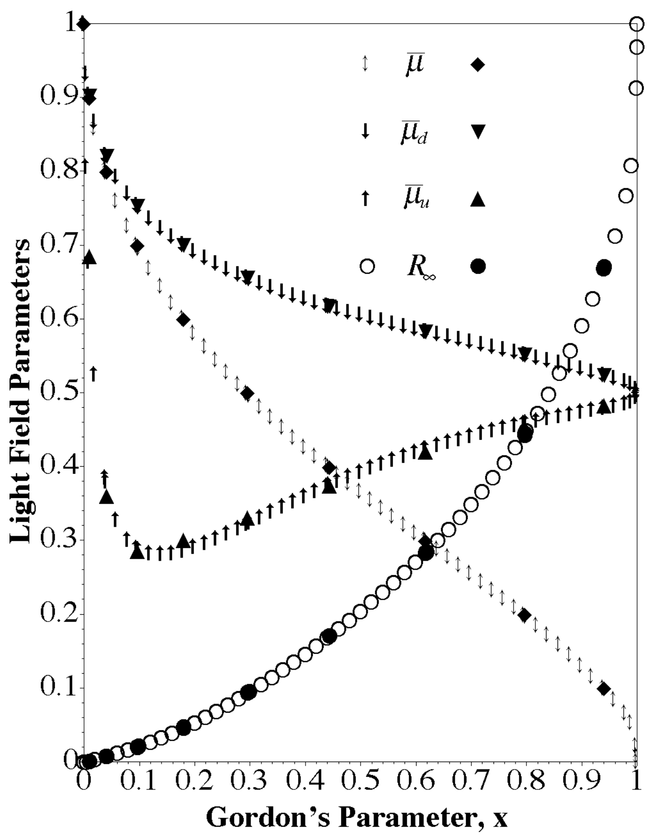

Fig.1 Dependencies of the integral light field parameters $\bar\mu$, $\bar\mu_d$, $\bar\mu_u$, and R_∞ on the parameter x. The filled symbols denote experimental values, while others correspond to computed ones.

fluxes of diffuse radiation (without halo)

$$E_d(\tau)=[E_0+M(\tau)]e^{-\alpha_\infty\tau}+R_0\,N(\tau)\left(e^{\alpha_0\tau}-e^{-\alpha_\infty\tau}\right), \quad (39)$$

$$E_u(\tau)=R_\infty[E_0+M(\tau)]e^{-\alpha_\infty\tau}+N(\tau)\left(e^{\alpha_0\tau}-R_0\,R_\infty\,e^{-\alpha_\infty\tau}\right) \quad (40)$$

where

$$M(\tau)=\left(1-R_0\,R_\infty\right)^{-1}\int_0^\tau\left\{\left[f_1(\tau')+R_0\,f_2(\tau')\right]e^{\alpha_\infty\tau'}\right. \\ \left.-R_0\left[R_\infty\,f_1(\tau')+f_2(\tau')\right]e^{-\alpha_0\tau'}\right\}d\tau', \quad (41)$$

$$N(\tau)=\left(A_B-R_\infty\right)\left(R_0\,\Delta_H\right)^{-1}\left[E_0+M(\tau_H)\right]e^{-\nu\tau_H}+ \\ \left(1-R_0\,R_\infty\right)^{-1}\int_\tau^{\tau_H}\left[R_\infty\,f_1(\tau')+f_2(\tau')\right]e^{-\alpha_0\tau'}d\tau'+ \quad (42) \\ A_B\left(R_0\,\Delta_H\right)^{-1}\int_0^{2\pi}d\varphi\int_0^1 L_q(\mu,\varphi)\exp\left[-(\alpha_0+\alpha/\mu)\,\tau_H\right]\mu\,d\mu,$$

$$\Delta_H=R_0^{-1}-A_B+(A_B-R_\infty)\,e^{-\nu\tau_H}, \\ \nu=\alpha_0+\alpha_\infty\equiv x\left(R_\infty^{-1}-R_0\right)/\mu_d\,. \quad (43)$$

For totally diffuse illumination of the medium we assume that the light flux from external sources passing through the upper boundary

$$E_q^0=E_d^f(0)\equiv\int_0^{2\pi}d\varphi\int_0^1 L_q(\mu,\varphi)\,\mu\,d\mu \quad (44)$$

is completely diffuse and we take it into account by the boundary condition $E_u^0(0)=E_q^0$, while we set $f_i(\tau)$ equal to zero. In addition, making the substitutions $E_0=E_q^0$, $M(\tau)=0$, $N(\tau)=(A_B-R_\infty)\,E_q^0\,(R_0\,\Delta_H)^{-1}e^{-\nu\tau_H}$ in (35), we obtain

$$E_d(\tau)=E_q^0\Delta_H^{-1}\left\{\left(R_0^{-1}-A_B\right)\exp(-\alpha_\infty\tau\,)\right. \\ \left.+(A_B-R_\infty)\exp\left[-\alpha_0(\tau_H-\tau)-\alpha_\infty\tau_H\right]\right\}, \quad (45)$$

$$E_u(\tau)=E_q^0\Delta_H^{-1}\left\{R_\infty\left(R_0^{-1}-A_B\right)\exp(-\alpha_\infty\tau\,)\right. \\ \left.+\left[(A_B-R_\infty)/R_0\right]\exp\left[-\alpha_0(\tau_H-\tau)-\alpha_\infty\tau_H\right]\right\}, \quad (46)$$

Now we calculate the transmittance of the layer $(0,\,\tau_H)$ for diffuse light $T(\tau)=E_d(\tau)/E_d(0)$ and the diffuse reflectance $R(\tau)=E_u(\tau)/E_d(\tau)$. Using (44)-(46), we obtain

$$T(\tau)=\frac{(R_0^{-1}-A_B)+(A_B-R_\infty)\,e^{-\nu(\tau_H-\tau)}}{(R_0^{-1}-A_B)+(A_B-R_\infty)\,e^{-\nu(\tau_H-\tau)}}\exp\left(-\alpha_\infty\tau\right), \quad (47)$$

$$R(\tau)=R_\infty\frac{(R_0^{-1}-A_B)+(A_B-R_\infty)(R_0\,R_\infty)^{-1}e^{-\nu(\tau_H-\tau)}}{(R_0^{-1}-A_B)+(A_B-R_\infty)\,e^{-\nu(\tau_H-\tau)}}\,. \quad (48)$$

The functional dependencies of R_∞, R_0, α_∞, α_0 upon the parameters of medium a and b_b are determined by (26)-(29), (31), (32), and (36).

In the limiting case of optically thick layer $(H>>[(\alpha_0+\alpha_\infty)(a+b_b)]^{-1}, R\to R_\infty)$ it is possible to express parameter $x=b_b/(a+b_b)$ in terms of the experimentally measurable quantity R_∞ Using the data of Table 1 and the asymptotic conditions given by (25), we get the following empirical equation

$$x=1-(1-R_\infty)^2\left\{r_1-r_0\,(1-R_\infty)\right. \\ \left.+(1-R_\infty)^2\left[1+r_1-\sum_{n=2}^{6}r_n\,R_\infty^{\frac{n-3}{4}}\right]\right\}. \quad (49)$$

The coefficients r_n are given in Table 2. The correlation coefficient between the calculated via (49) values and experimental data exceeds 0.99.

CONCLUSION

Our method of calculation is found to produce results with accuracies in the range of 15% for all types of natural water optical situations (open ocean to coastal environments). Comparison with Monte-Carlo simulations shows that (49) can be used for processing remotely sensed data collected over coastal and open ocean areas with the same 15% precision.

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