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AN ALGORITHM FOR REMOTE RADAR REAL-TIME ASSESSMENT OF THE ROUGHNESS OF EARTH'S NATURAL SURFACES*

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ABSTRACT

An analysis of the geometrical factors involved in the calculation of a microwave pulse return in the remote sounding of Earth's natural surfaces is given and an algorithm for processing such data is proposed. In this approach the microwave signal reflected from the surface is expressed as an integral which includes all geometrical factors and the physical response of the underlying medium to the pulse radiation. The problem of radar sounding of the Earth is reduced to the calculation of the medium response to narrow pulse irradiation of its surface which can be resolved using electromagnetic or radiative transport approaches. The resulting radar equation expresses the intercepted signal as a function of the sounding geometry, the radar pulse shape, the Earth surface roughness, and the physical characteristics of the medium. These characteristics are represented by a Green function or the response of this medium to illumination by a very narrow pulse.

1.0 INTRODUCTION

The scientific and practical importance of microwave remote study of the Earth's natural covers has been discussed in detail in a number of papers (Foster, Hall, and Chang 1987; Brooks, Campbell, Ramseier, Stanley and Zwally, 1978). The problem of interpretation and deciphering of a microwave pulse signal reflected by snow-and-ice surfaces has not so far been resolved (Vickers, 1971; Zwally, Major, Brenner and Binschadler, 1971). This fact is not surprising since the solution to the less difficult problem of the reflection of a narrow electromagnetic beam, falling under arbitrary angle on the scattering surface and absorbing medium, has not yet been obtained in its general form. A number of publications on this subject are available (Zege, Ivanov and Katsev, 1991). Most of them deal with optical beams. Unfortunately, the simplest solutions to the narrow pulse reflection problem have an analytical form only in Fourier space and their functional dependence on the space and time coordinates has a cumbersome integral form. The problem of finding the microwave pulse return from the Earth surface is more difficult because a radar radiates spherical waves whose angular dependence is determined by the radar gain.

At each particular moment in time a radar receiver records the signal reflected from the Earth surface integrated over a ring-shaped footprint area. The diameter and, consequently, the total area of the ring footprint varies with time. This effect leads to the distortion of the time envelope of the received microwave signal.

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In this work we have expressed the microwave pulse signal reflected from the Earth's surface in the form of an integral. The integrand of this expression depends on *geometrical factors* and *the physical response* of the underlying medium to the direct pulse irradiation. Therefore, the problem of radar pulse sounding of the Earth's cover is reduced to the problem of calculating the response of a medium to narrow pulse irradiation of its surface.

2.0 DERIVATION OF THE MAIN EQUATION

Consider a radar installed on a satellite in an orbit situated at a distance H from the Earth's surface. We set up a Cartesian coordinate system with the $0z$-axis directed from the center of the radar antenna to the Earth at a normal angle to its surface. We define $\mathbf{n}_0 = (\sqrt{1-\mu_0^2}, 0, \mu_0)$ as a unit vector oriented in the direction of maximum intensity of the signal radiated by the radar antenna, $\mu_0 = \cos\theta_0$, $\theta_0 << 1$. For simplicity we consider an infinitely short impulse signal of the form

$$P_{rad}(\mathbf{n},t) = P_0\, G(\mathbf{n}\mathbf{n}_0)\, \delta(t), \qquad (1)$$

where $\mathbf{n} = (\sqrt{1-\mu^2}\cos\varphi,\ \sqrt{1-\mu^2}\sin\varphi,\ \mu)$ is a unit vector in the direction of sounding, $\mu = \cos\theta$ with θ as zenith angle, φ is the azimuth angle of sounding, $\delta(t)$ is the Dirac delta function (Morse and Feshbach, 1953) and $G(\mathbf{n}\mathbf{n}_0)$ is a gain which determines the angular distribution of energy radiated by the radar. We assume the following Gaussian form for the gain:

$$G(\mathbf{n}\mathbf{n}_0) = \frac{1}{2\pi D}\exp\left(\frac{\mathbf{n}\mathbf{n}_0 - 1}{D}\right) \approx \frac{1}{2\pi D}\exp\left\{-\frac{\left[\cos^{-1}(\mathbf{n}\mathbf{n}_0)\right]^2}{2D}\right\}, \qquad D \equiv \frac{\langle \theta^2 \rangle}{2} << 1. \qquad (2)$$

The normalization of the gain is performed under the condition

$$\int G(\mathbf{n}\mathbf{n}_0)\, d\mathbf{n} \ \equiv\ \int_0^{2\pi} d\varphi' \int_0^1 G(\mu')\, d\mu' = 1. \qquad (3)$$

For satellite remote sounding radars the value $D \cong 10^{-4}$, which corresponds to an effective gain angle $\sqrt{\langle \theta^2 \rangle} \cong$0.8°=0.014 *rad*.

In what follows, we break up the derivation of our main result, Eqn. (14), into successive stages, each adopting a simplifying assumption. Should further improvement be necessary, each stage can separately be addressed.

2.1 STAGE 1: PULSE SHAPE AS A FUNCTION OF DISTANCE

A radar pulse signal which propagates in the atmosphere interacts with the air molecules and aerosol particles. As a result the pulse distorts in space and broadens in time. These transformations are described by the macroscopical Maxwell's equations (Skolnik, 1980; Landau and Lifshitz, 1984; Tatarskii, 1961). In this work we restrict ourselves to the microwave frequencies for which electromagnetic properties of the atmosphere are close to the corresponding properties of free space, *e. g.* we neglect effects due to dispersion, absorption and scattering. In this case Maxwell's equations can be reduced to the simple wave equation (Landau and Lifshitz, 1984), a solution of which at range R from the antenna for the source function (1) has the form

$$P_R(\mathbf{n},t) = P_0\, G(\mathbf{n}\mathbf{n}_0)\, \delta(t - R/c). \qquad (4)$$

2.2 STAGE 2: SIGNAL REFLECTED FROM THE SURFACE

Let us consider an elementary surface with an area dS in the footprint region. The angular coordinates of this surface are determined by the vector $\mathbf{n}$ (*e.g.* by the angles $\theta = \cos^{-1}\mu$ and μ). We denote $\mathbf{n}_s$ as a unit vector normal to this elementary surface and directed towards the atmosphere. Let us denote the projection of the elementary surface area on the horizontal plane as $dS_{xy} = \mathbf{n}_s\mathbf{n}_p \, dS$, where $\mathbf{n}_p$ is a unit vector normal to the horizontal plane, and the elevation of the elementary surface from the average level of the Earth in the footprint region as h. In this case the energy of the microwave radiation which falls on the elementary surface dS will be

$$dP_s = P_R \, d\Omega_s \equiv \frac{P_R}{R^2} dS(-\mathbf{n}_s\mathbf{n}) \equiv \frac{P_R\mu}{R^2} dS_{xy} = P_0 \, G(\mathbf{nn}_0)\,\delta(t - \frac{R}{c})\, d\Omega_s \,, \tag{5}$$

where $d\Omega_s = d\varphi \, d\mu \equiv d\varphi \sin\theta \, d\theta$ is a solid angle differential element and $R = (H-h)/\mu$ is the range from the antenna to the elementary surface dS.

We denote as $G_s(\mathbf{nn}_s, t)$ the Green function which is the medium response to an infinitely narrow, infinitely short unit power microwave pulse directed at an angle $\cos^{-1}(\mathbf{nn}_s)$ to the normal to the elementary surface. In this case the signal reflected from the surface will be

$$dP_s^B(t) = P_0 \, G(\mathbf{nn}_0) \; G(\mathbf{nn}_s, \; t - \frac{R}{c})\, d\Omega_s . \tag{6}$$

2.3 STAGE 3: SIGNAL FROM ELEMENTARY SURFACE INTERCEPTED BY RADAR

The signal from the elementary surface dS detected by the antenna will be

$$dP_{As}^B(t) = dP_s^B\left(t - \frac{R}{c}\right) d\Omega_A = \frac{A_e \, \mathbf{nn}_0}{R^2} \, dP_s^B\left(t - \frac{R}{c}\right), \tag{7}$$

where A_e is the receiving effective area of the radar antenna. According to Skolnik (1980) it is equal to

$$A_e = \frac{\lambda^2}{4\pi} P_0 \, G(\mathbf{nn}_0), \tag{8}$$

where λ is the wavelength of the microwave radiation. Consequently

$$dP_{As}^{B}(t) = \frac{\lambda^2}{4\pi R^2} G^2(\mathbf{nn}_0) G_s\left(\mathbf{nn}_s, t - \frac{2R}{c}\right) \mathbf{nn}_0 \, d\Omega_s , \tag{7}$$

where we introduce a time delay R/c, which can be obtained after solving the wave equation with a source function given by Eqn. (6).

After integration over the solid angle $d\Omega_s$ we obtain the signal intercepted by the radar antenna:

$$P_B^{\delta}(t) = \frac{\lambda^2 P_0}{4\pi} \int_{4\pi} \frac{\mu^2 G^2(\mathbf{nn}_0)\mathbf{nn}_0}{\left[H - h(\Omega_s)\right]^2} G_s\left(\mathbf{nn}_s(\Omega_s), t - \frac{2H}{\mu c} + \frac{2h}{\mu c}\right) d\Omega_s, \tag{10}$$

2.4 STAGE 4: INTEGRATION OF INTERCEPTED SIGNAL OVER RANDOM SURFACE

In Eqn. (10) we have shown the formal dependence of the elementary surface height $h(\Omega_s)$ and a normal to it $\mathbf{n}_s(\Omega_s)$ on the angles $\Omega_s \equiv (\mu,\varphi)$ over which this surface is observed. Actually the values h and $\mathbf{n}_s$ should be considered as independent (of Ω_s) random values. Because the radar receiver integrates signals from the immense number of elementary surfaces situated in a ring footprint and further processing usually involves averaging over a number of pulses (100, for example), which is equivalent to averaging over footprints, we could replace the Green function G_s in Eqn. (10) by the following average Green function:

$$\tilde{G}_s\left(\mu, t - \frac{2H}{\mu c}\right) = \left\langle G_s(\mathbf{nn}_s, t - \frac{2H}{\mu c} + \frac{2h}{\mu c})\right\rangle_{h,\mathbf{n}_s} \equiv \int f(h) \int \tilde{F}(\mathbf{n}_s) G_s\left(\mathbf{nn}_s, t - \frac{2H}{\mu c} + \frac{2h}{\mu c}\right) dh\, d\mathbf{n}_s, \tag{11}$$

where $f(h)$ is the height distribution function of the elementary surfaces (Bass and Fuks, 1979) and $\tilde{F}(\mathbf{n}_s)$ is the distribution function of these surfaces over orientation angles. These functions are normalized according to the following conditions:

$$\int f(h)\, dh = 1, \qquad \int \tilde{F}(\mathbf{n}_s)\, d\mathbf{n}_s = 1\,. \tag{12}$$

Using the fact that $h << H$ and also taking into account Eqn. (11), we obtain

$$P_B^{\delta}(t) = \frac{\lambda^2 P_0}{4\pi H^2} \int_0^{2\pi} d\varphi \int_0^1 \mu^2\, \mathbf{nn}_0\, G^2(\mathbf{nn}_0)\, \tilde{G}_s\left(\mu, t - \frac{2H}{\mu c}\right) d\mu. \tag{13}$$

After inserting into Eqn. (13) the functional dependence of the gain on $\mathbf{nn}_0$ given by Eqn.(2) and integrating over azimuth angle φ, taking into account that $\langle \theta^2 \rangle^2 << 1$, $\theta_0 << 1$, we obtain

$$P_B^{\delta}(t) = \frac{\lambda^2 P_0}{2\pi^2 \langle \theta^2 \rangle^2 H^2} e^{-\frac{2\theta_0^2}{\langle \theta^2 \rangle}} \int_0^1 I_0\left(\frac{4\theta_0 \sqrt{1-\mu^2}}{\langle \theta^2 \rangle}\right) \tilde{G}_s\left(\mu, t - \frac{2H}{\mu c} + \frac{2h}{\mu c}\right) e^{-\frac{4(1-\mu)}{\langle \theta^2 \rangle}} \mu^2\, d\mu, \tag{14}$$

where $I_0(x) = \sum_{s=0}^{\infty} x^{2s} / \left[2^{2s} (s!)^2\right]$ is the zero-order modified Bessel's function (Arfken, 1985).

2.5 STAGE 5: GENERAL TIME ENVELOPE FUNCTION FOR SOUNDING PULSE

In the case when the time envelope of the sounding pulse is described by a function $\tilde{E}(t)$, $\left(\int \tilde{E}(t)\,dt = 1\right)$, due to the linearity of Maxwell's equations, the detected microwave signal will be a convolution of $\tilde{E}(t)$ with $P_B^{\delta}(t)$ given by Eqn.(14), *i.e.*

$$P_B(t) = \tilde{E}(t) \otimes P_B^{\delta}(t) \equiv \int_0^{\infty} \tilde{E}(t-t')\,P_B^{\delta}(t')\,dt'. \qquad (15)$$

Consequently, Eqn. (14) reduces the problem of calculating the time envelope of the radar pulse reflected from the natural surface to the problem of finding the Green function of the medium illuminated by an infinitely narrow short microwave pulse.

Some success in solving similar problems has been achieved in optics in the works dealing with the calculation of the functional dependence of a light pulse reflected from scattering and absorbing media (Zege, Ivanov and Katsev, 1991). Because the depth of penetration of microwave radiation with frequencies of the order of 14 GHz $\{\lambda \approx 2\,cm\}$ into snow and ice covers is much greater than the wavelength λ, we can use optical methods and, in particular, radiative transfer theory, for calculating the microwave pulse return from these surfaces.

3.0 EXAMPLES OF RADAR PULSE RETURN FROM SURFACES WITH A SIMPLE GREEN FUNCTION

Here we restrict ourselves to a simple form for the Green function. We consider the radar pulse reflection from rough surfaces made up of highly absorptive substances, such as very saline broken ice, which are not specularly reflecting. For these cases $\tilde{F}(\mathbf{n}_s) = 1/2\pi$, and the mean Green function will be given by the following equation

$$\tilde{G}_s^A\left(\mu, t - \frac{2H}{\mu c}\right) = \frac{A_m}{\pi}\int_{-\infty}^{+\infty} \delta\left(t - \frac{2H}{\mu c} + \frac{2h}{\mu c}\right) f(h)\,dh, \qquad (16)$$

where A_m is the surface albedo with respect to microwave radiation: $A_m = 2\int_0^1 \rho_m(\mu)\,\mu\,d\mu$, and $\rho_m(\mu)$ is the reflection coefficient of a non-polarized microwave plane wave falling on the surface at an angle $\cos^{-1}\mu$.

By introducing $t_H = 2H/c$, $t_s = t - t_H$ and integrating Eqn. (16) over h, we have:

$$\tilde{G}_s(t_s) = \frac{\mu c A_m}{2\pi} f\left(H(1-\mu) - \frac{\mu c t_s}{2}\right). \qquad (17)$$

After inserting Eqn. (17) into Eqn. (14) we get:

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$$P(\tau) = h_0\, e^{-2(\tau+q)} \int_a^1 f(h_0 y)\, I_0\left(4\sqrt{q(\tau+py)}\right) e^{2py}\, dy, \tag{18}$$

where $P(\tau) = P_B^{\delta}(\tau)/P_0^*$, $\tau = t_s/t_G = (t-t_H)/t_G$, $t_G = H\langle\theta^2\rangle/c =$ 520 $nsec$, $P_0^* = A_m P_0\left[c\lambda^2 \Big/ \left(4\pi^3 \langle\theta^2\rangle^2 H^3\right)\right]$ (for $\lambda =$ 2.14 cm, $\sqrt{\langle\theta^2\rangle} = 0.8^o \approx 0.014\, rad$, $H =$ 800 km, $P_0^* \approx 5.63\ 10^{-17}\ sec^{-1} \times A_m P_0$); h_0 is the maximum amplitude of the function $f(h)$ $q = \theta_0^2/\langle\theta^2\rangle$, $p = 2h_0 \big/ \left(H\langle\theta^2\rangle\right)$ and $a = -\min(1, \tau/p)$. When deriving Eqn. (18) we have taken into account that $f(h) = 0$ when $|h| \ge h_0$. Let us calculate $P(\tau)$ for the three simple cases.

3.1 VERY SALINE BROKEN ICE WITH SMALL-SCALE ROUGHNESSES $h_0 << \lambda$ (DULL EVEN SURFACE WITHOUT SPECULARLY REFLECTING PARTS).

In this case we can accept the approximation: $f(h) = \delta(h)$, and Eqn. (18) yields

$$P_D(\tau) = I_0\left(4\sqrt{q\tau}\right) \exp[-2(\tau+q)]. \tag{19}$$

3.2 VERY SALINE BROKEN ICE WITH ARBITRARY-SCALE ROUGHNESS.

This case can be represented with the following distribution function:

$$f(h) = \begin{cases} h_0^{-1} \cos^2(\pi h/2h_0), & |h| < h_0, \\ 0\quad, & |h| \ge h_0. \end{cases} \tag{20}$$

Substituting this distribution function into Eqn. (18), we get

$$P_h(\tau) = e^{-2(\tau+q)} \int_a^1 \cos^2\frac{\pi y}{2}\, I_0\left(4\sqrt{q(\tau+py)}\right) e^{-2py}\, dy, \tag{21}$$

where $a = -\min(1, \tau/p)$.

Figures 1-3 show the time dependence of the radar signal reflected from these surfaces. Calculations have been made for the following viewing angles: θ_0 = 0°, 0.2°, 0.4°, 0.6°, 0.8°, 1° and 1.2°, and roughness amplitudes h_0 = 0 m, 0.3 m, 0.9 m, 2.7 m and 8.1 m, which correspond to time delays of $t_0 = 2h_0/c =$ 0 $nsec$, 3 $nsec$, 6 $nsec$, 18 $nsec$ and 54 $nsec$.

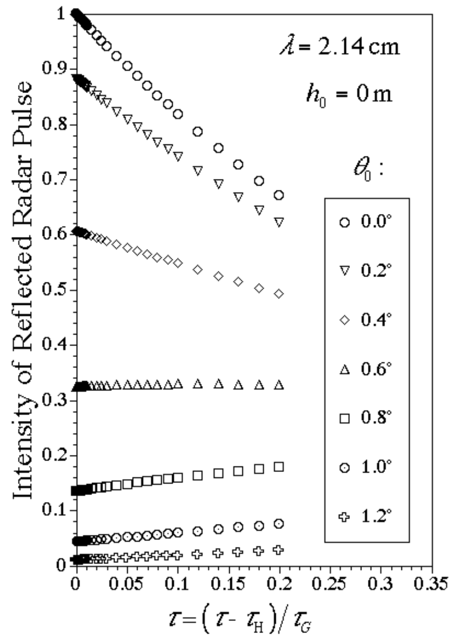

Fig. 1. Radar pulse return from an even, dull surface made of a highly absorbing substance for different viewing angles θ_0.

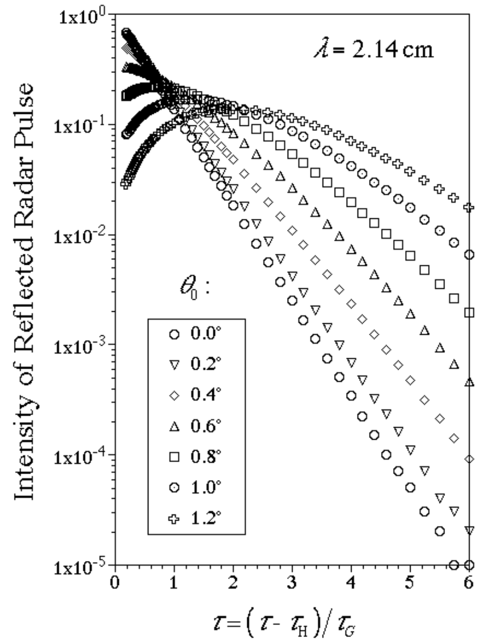

Fig. 2. The same as Fig. 1, only for large τ.

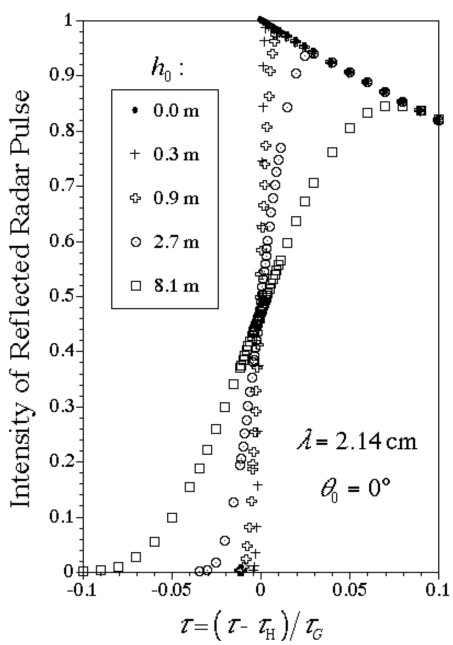

Fig. 3. The transformation of the reflected pulse front with respect to different scales of roughness h_0.

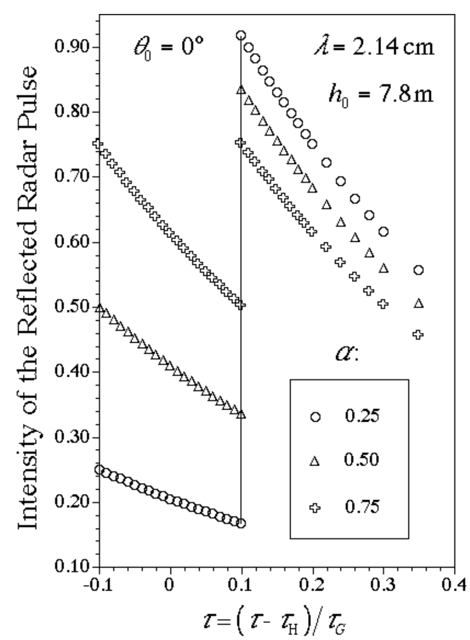

Fig. 4. Radar Pulse response from two-level surfaces with different values of parameter α.

3.3 TWO-LEVEL SURFACE OF VERY SALINE ICE WITHOUT SPECULARLY REFLECTING SURFACES AND WITH SMALL-SCALE ROUGHNESS ($h_0 << \lambda$).

We represent the distribution function of the two-level surface in the following form:

$$f(h) = \alpha\,\delta(h-h_0) + (1-\alpha)\,\delta(h+h_0), \qquad (22)$$

where α is that portion of the total surface which constitutes the higher level. In this case the radar pulse response can be expressed by the equation

$$P(\tau) = e^{-2(\tau+q)}\left[\alpha\,\theta(\tau+p)e^{-2p}I_0\left(4\sqrt{q(\tau+p)}\right) + (1-\alpha)\theta(\tau-p)e^{2p}I_0\left(4\sqrt{q(\tau-p)}\right)\right]. \quad (23)$$

Illustrations of this case are shown in Figs. 4-6.

We do not see major difficulties for calculating the radar pulse response from moderately absorbing and scattering substances and from surfaces with a specular reflection. For the first case, instead of a Dirac's delta function in Eqn.(16) we have to use the exact expression for the time dependence of the reflected pulse, which will depend on the absorption and scattering properties of the medium. These values are functionals of the characteristics of particles, such as their complex dielectric permittivities, their fractional concentrations, and their size distribution functions. For the second case we have to take into account the highly anisotropic behavior of the Green function. This behavior will result in the appearance of very narrow (on the order of several nanoseconds) peaks at the values of $\tau = 0$ (shown in figures 2 and 3) in the responses. These peaks correspond to the location of the specularly reflecting surfaces.

We have shown that only in the first and third cases (the simplest ones) of salted ice with small-scale surface roughness can the reflected microwave pulse registered by the radar receiver be expressed by an analytical expression. And this can only be done when the sounding signal has a delta-function time envelope. In the more complex case of broken saline ice with the simplest approximation of the average Green function (case 3.2) the result is given by an integral over the product of exponential, Bessel's and height distribution functions. In the cases of complex, many-layered and uneven snow-and-ice surfaces, expressions for the reflected microwave pulse should be much more cumbersome.

4.0 CONCLUSION

We have derived an analytical expression, Eqn. (14), which connects the value of a pulse microwave signal, reflected from a statistically random reflecting snow-and-ice surface and detected by radar, with the sounding geometrical parameters θ_0, $\langle\theta_0^2\rangle$ and with the characteristics of the sounding medium $\tilde{G}_s$. Under the assumptions of a transparent atmosphere this equation is valid for narrowly-directed $\left(\sqrt{\langle\theta^2\rangle} << 1\right)$ microwaves characterized by a Gaussian-gain shape, close to nadir viewing angles $(\theta_0 << 1)$ and statistically independent characteristics of the underlying surface. We have not considered here time-dependent fluctuations of the surface characteristics (Anonymous, 1946). In the case of satellite sounding over a large observation area

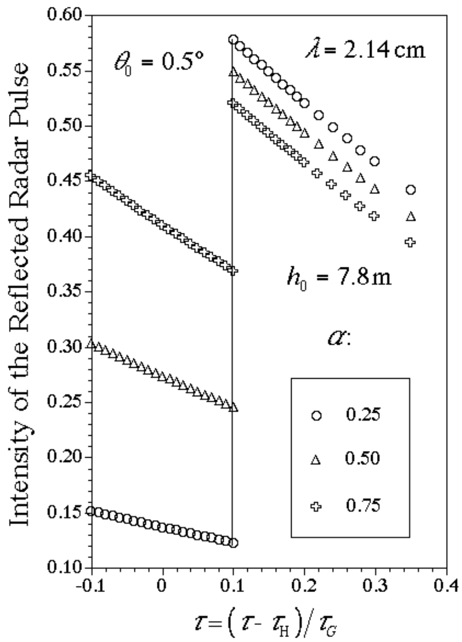

Fig. 5. Radar Pulse response from two-level surfaces with different values of parameter α.

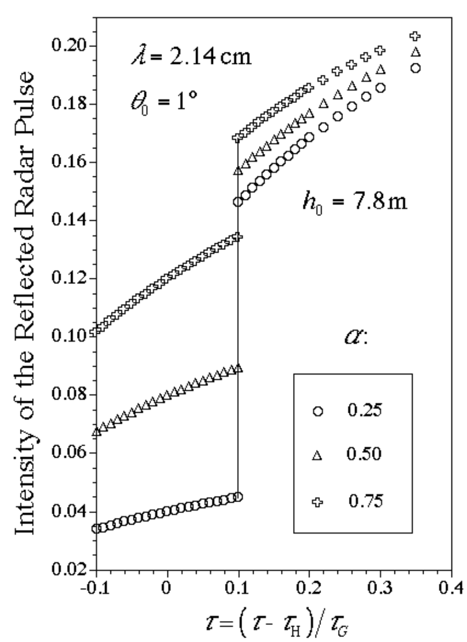

Fig. 6 Radar Pulse response from two-level surfaces with different values of parameter α.

they can be neglected. The derivation of Eqn. (14) was made in successive stages, each adopting a simplifying assumption. The approach used here permits us to separately improve every stage of the calculations if it proves to be necessary.

We believe that in the sense of achieving better precision in the calculation of return signals and developing algorithms for the interpretation of the forms of these signals the most important stage for improvement is the ground stage. It can consist of two parts:

1) developing electromagnetic models of the snow-and-ice covers suitable for use in radiative transfer problems; and,

2) developing theories of reflection of the narrow electromagnetic beams from snow-and-ice layers.

The very concise and simplified analysis of radar pulse propagation made here convinces us that the final solution to the problem of creating software for the processing and interpretation of spaceborne radar sounding of snow-and-ice surfaces is a very complex and labor-consuming goal, but an achievable one.

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