

LIGHT SCATTERING COEFFICIENT OF WATER AT CONCENTRATIONS OF HYDROSOLS TYPICAL FOR LAKES AND SHALLOW MARINE WATERS

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ABSTRACT

The scattering coefficient of water as a function of concentration of hydrosol particles is calculated. The approach used is based on the Maxwell's equations in a stochastically scattering water. The water is modeled as thermally fluctuated medium filled with the hydrosol particles. It is found that the scattering coefficient quadratically depends on concentration when the concentration of scatterers is very small. The scattering coefficient is linear to concentration of scatterers at values typical to the open ocean. At the values of concentrations typical to coastal waters the dependence on concentration weakens and reaches the saturation at very high values.

INTRODUCTION

In a majority of publications available today the light scattering coefficient by water is considered as linearly dependent on concentrations of hydrosols. The experiments by Prieur and Sathyendranath (1981) show that at certain concentrations of chlorophyll C_c , typical to coastal waters, the dependence of water absorption coefficient on C_c is nonlinear. Clark and Backer (1980) showed that the scattering coefficient of water is non-linearly dependent on chlorophyll concentration that is strongly correlated to the concentration of scattering matter of biologic origin.

In this paper an attempt is made to develop an approach to calculate the scattering coefficient of water as a function of concentration of hydrosol particles C_p . The approach is based on the solutions of the Maxwell's equations in a stochastically scattering medium (water). The water is modeled as thermally fluctuated medium filled with the hydrosol scatterers.

The result of this paper are Eqns. (46)-(47) for the water scattering coefficient. The scattering coefficient b is linear with concentration of scattering particles at values typical to the open ocean. The coefficient quadratically depends on the concentration when the concentration is very small (typical for the Sargasso Sea waters). At the concentrations close to the values that are typical to coastal waters, the dependence on concentrations weakens and reaches saturation at very high values. The results of this paper can explain some experimental data obtained in turbid coastal waters.

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APPROACH

The radiative transfer theory, predominantly used in ocean optics, cannot explain nonlinear dependence of inherent optical properties on concentration of scatterers (Haltrin, 1985; Haltrin and Kattawar, 1993; Haltrin, and Weidemann, 1996; Haltrin, 1997). To investigate this problem we should start from the Maxwell's equations in stochastically scattering medium. The mathematical formalism of scattering in stochastic medium is identical to the formalism of quantum statistical mechanics (Gould, 1967; Frish, 1968). The photons themselves are always quantum particles. For these reasons we have chosen to use the quantum-mechanical statistical approach formulated by Abrikosov, Gorkov and Dzyaloshinski (1963).

In this paper the scattering coefficient on hydrosol particles is calculated through the dielectric permittivity of the hydrosol component in the water. The dielectric permittivity is a constituent part of the Fourier transform of the Green's function of the Maxwell's equations in water. By definition, the Green's function is a solution of these equations when a source function is assumed as an infinitely short and localized at one point light pulse (Morse and Feshbach, 1953; Haltrin, 1977; Haltrin, 1980).

INTERACTION HAMILTONIAN

Let us use the standard method of the derivation of the interaction Hamiltonian. The change in the energy E of water after injunction of hydrosol particles is

$$\delta E = (4\pi)^{-1} \int d^3r \mathbf{E}(\mathbf{r}, t) \delta \mathbf{D}(\mathbf{r}, t), \quad (1)$$

here $\mathbf{E}(\mathbf{r}, t)$ is the electric field, $\delta \mathbf{D}(\mathbf{r}, t)$ is the change in the electric induction $\mathbf{D}(\mathbf{r}, t)$ related to the difference of the dielectric permittivity of hydrosol particles from the dielectric permittivity of surrounding water, \mathbf{r} is a spatial coordinate, t is time. The electric induction inside hydrosol particle is (Landau and Lifshitz, 1960):

$$\mathbf{D}_m(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + \int_0^\infty f_m(t') \mathbf{E}(\mathbf{r}, t - t') dt', \quad (2)$$

here m denotes a number of the particle, the function f_m determines the dispersion properties of the m^{th} hydrosol particle. For further convenience let us introduce the following Heavyside-type function (Morse and Feshbach, 1953):

$$H_m(\mathbf{r} - \mathbf{r}_m) = \begin{cases} 1, & \mathbf{r} \in V_m, \\ 0, & \mathbf{r} \notin V_m, \end{cases} \quad (3)$$

that is equal to one inside m^{th} particle and equal to zero outside (here \mathbf{r}_m is a vector that points to the gravity center of the m^{th} particle, V_m is the volume of the m^{th} particle). Using function H_m , let us write down the change in the energy of the water caused by injunction of hydrosols. The change in electric induction is

$$\delta \mathbf{D}(\mathbf{r}, t) = \int_0^\infty \delta f(\mathbf{r}, t') \mathbf{E}(\mathbf{r}, t - t') dt', \quad (4)$$

$$\delta f(\mathbf{r}, t') = \sum_m \langle H_m(\mathbf{r} - \mathbf{r}_m) \rangle_m [f_m(t') - f_w(t')], \quad (5)$$

here $f_w(t)$ is a dispersion function of the water without hydrosols. The angular brackets $\langle \dots \rangle_m$ denote averaging over all possible positions of \mathbf{r}_m . The summation is made over all hydrosol particles in the water volume V_0 .

The distribution function of hydrosol particles can be represented as a series over the powers of water density

$$F(\mathbf{r}, t) \equiv \alpha \rho(\mathbf{r}, t) + (\beta/2) \rho^2(\mathbf{r}, t) + \dots, \quad \int F(\mathbf{r}, t) d^3 r = 1. \quad (6)$$

Let us keep only the first term in (6), then

$$F(\mathbf{r}, t) \equiv \rho(\mathbf{r}, t)/(V_0 \rho_0) = V_0^{-1} + \delta\rho(\mathbf{r}, t)/(V_0 \rho_0), \quad (7)$$

here $\rho_0 = V_0^{-1} \int \rho(\mathbf{r}, t) d^3 r$ is the average water density in the volume V_0 , $\delta\rho(\mathbf{r}, t) = \rho(\mathbf{r}, t) - \rho_0$ is the fluctuation of the water density. So, we have

$$\langle H_m(\mathbf{r} - \mathbf{r}_m) \rangle_m = \int F(\mathbf{r}', t) H_m(\mathbf{r} - \mathbf{r}') d^3 r' = C_m + (V_0 \rho_0)^{-1} \int \delta\rho(\mathbf{r}, t) H_m(\mathbf{r} - \mathbf{r}') d^3 r', \quad (8)$$

here $C_m = V_m / V$. In this case

$$\delta f(\mathbf{r}, t') = \sum_m C_m [f_m(t') - f_w(t')] + \rho_0^{-1} \int \delta\rho(\mathbf{r}', t') v(\mathbf{r} - \mathbf{r}', t') d^3 r', \quad (9)$$

$$v(\mathbf{r}, t) = V_0^{-1} \sum_m H_m(\mathbf{r}) [f_m(t) - f_w(t)]. \quad (10)$$

For the changes in the electric induction we have,

$$\delta \mathbf{D}(\mathbf{r}, t) = \delta \mathbf{D}_0(\mathbf{r}, t) + \delta \mathbf{D}_s(\mathbf{r}, t), \quad (11)$$

here

$$\delta \mathbf{D}_0(\mathbf{r}, t) = \sum_m C_m \int_0^\infty dt' [f_m(t') - f_w(t')] \mathbf{E}(\mathbf{r}, t - t'), \quad (12)$$

$$\delta \mathbf{D}_s(\mathbf{r}, t) = \rho_0^{-1} \int_0^\infty dt' \int d^3 r' \delta\rho(\mathbf{r}', t') v(\mathbf{r} - \mathbf{r}', t - t') \mathbf{E}(\mathbf{r}, t - t'). \quad (13)$$

The first correction term in Eqn. (11) is not connected with scattering. It determines correction to the water dielectric permittivity due to introduction of hydrosols. Later we incorporate this term into the original water dielectric permittivity by renormalizing its value.

When the volume concentration of hydrosol particles is small ($C_V \equiv \sum_m C_m \ll 10^{-3}$) it is possible to neglect dispersion properties of particles. In this case we have

$$f_m(t) - f_w(t) \equiv (\varepsilon_m - \varepsilon_{0w}) \delta(t + 0). \quad (14)$$

Here ε_m is the dielectric permittivity of the m^{th} hydrosol particle, ε_{0w} is the dielectric permittivity of the water without particles, and $\delta(t)$ is the Dirac's delta. Taking into account Eqn. (14), we have

$$v(\mathbf{r}, t) = \delta\varepsilon_h C_V \Delta(\mathbf{r}) \delta(t + 0), \quad (15)$$

$$\delta\varepsilon_h = C_V^{-1} \sum_m (\varepsilon_m - \varepsilon_{0w}) C_m = C_V^{-1} \sum_m \varepsilon_m C_m - \varepsilon_{0w} = \bar{\varepsilon}_h - \varepsilon_{0w}, \quad (16)$$

$$\Delta(\mathbf{r}) = (\delta\varepsilon_h C_V V_0)^{-1} \sum_m (\varepsilon_m - \varepsilon_{0w}) H_m(\mathbf{r}), \quad \int \Delta(\mathbf{r}) d^3 r = 1. \quad (17)$$

Let us express water density fluctuation through the parameters of the phonon field. From the continuity equation

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \text{div}[\dot{\mathbf{q}}(\mathbf{r}, t) \rho(\mathbf{r}, t)] = 0, \quad (18)$$

here $\dot{\mathbf{q}}(\mathbf{r}, t)$ is a fluctuation coordinate. In the first approximation over $\delta\rho(\mathbf{r}, t)$ (Abrikosov, Gorkov, Dzyaloshinski, 1963) we have the following equation

$$\frac{\delta\rho(\mathbf{r}, t)}{\rho_0} = -\text{div} \mathbf{q}(\mathbf{r}, t) = -\frac{\varphi(\mathbf{r}, t)}{u_0 \sqrt{\rho_0}}, \quad (19)$$

here u_0 is a velocity of the thermal fluctuation in water (with the linear phonon dispersion law: $\omega_0(\mathbf{k}) = u_0 |\mathbf{k}|$, where \mathbf{k} is the phonon's momentum), $\varphi(\mathbf{r}, t)$ is a function that determines a high-frequency phonon field. By substituting Eqns. (13), (15) and (17) into Eqn. (1), we have for the change of energy

$$\delta E = -\frac{\delta \varepsilon_h C_V}{4\pi u_0 \sqrt{\rho_0}} \int d^3 r \mathbf{E}(\mathbf{r}, t) \int d^3 r' \Delta(\mathbf{r} - \mathbf{r}') \varphi(\mathbf{r}', t) \mathbf{E}(\mathbf{r}', t). \quad (20)$$

Let us express electric field $\mathbf{E}(\mathbf{r}, t)$ through the potentials of electromagnetic field

$$\mathbf{E}(\mathbf{r}, t) = -\frac{1}{c} \frac{\partial \mathbf{A}(\mathbf{r}, t)}{\partial t} - \text{grad} \Phi(\mathbf{r}, t). \quad (21)$$

Here \mathbf{A} is the vector potential, and Φ is the scalar potential, c is the speed of light. We accept the calibration with $\Phi = 0$. Hydrosol particles in this model are perceived as small potentials in a Brownian motion. They are characterized by the size distribution function: $\varphi(a)$ ($\int_0^\infty \varphi(a) da = 1$). Such a model allows one to derive the operator of thermal density fluctuations $\hat{\psi}$. Let us introduce the following dimensionless parameters,

$$\psi(\mathbf{r}, t) = \frac{1}{4\pi} \sqrt{\frac{\bar{a}^3}{T}} \int d^3 r' \Delta(\mathbf{r} - \mathbf{r}') \varphi(\mathbf{r}', t), \quad \Delta(\mathbf{r}) = \frac{\int_r^\infty \varphi(a) da}{4\pi \int_0^\infty r^2 dr \int_r^\infty \varphi(a) da} \quad (22)$$

$$g = \gamma C_V, \quad \gamma = \delta \varepsilon_h \sqrt{\frac{T}{\bar{a}^3 \rho u^2}} \approx 10^3 \div 10^5 \quad (23)$$

here $\bar{a} = \int_0^\infty \varphi(a) a da$ is a mean radius of hydrosol particles, T is the absolute temperature in Kelvins.

Now we have the formula for the interaction Hamiltonian between photons and thermal density fluctuations:

$$\hat{H}_{\text{int}}(t) = -\frac{g}{c^2} \int d^3 r \frac{\partial \hat{A}_\alpha(\mathbf{r}, t)}{\partial t} \hat{\psi}(\mathbf{r}, t) \frac{\partial \hat{A}_\alpha(\mathbf{r}, t)}{\partial t}, \quad \alpha = 1, 2, 3, \quad (24)$$

here \hat{A}_α is a photon field operator that corresponds to a vector potential of light wave, index α denotes vector's component. Repeating indices everywhere in this article imply summation.

GREEN'S FUNCTION

Let us write a Green's function of photons propagating in a nonscattering medium with the dielectric permittivity ε_0 . It can be represented as a sum of the transverse and longitudinal components (Landau and Lifshitz, 1960; Haltrin, 1980). In the energetic-momentum representation the Green's function has the following form:

$$D_{\alpha\beta}^{(0)}(\omega, \mathbf{k}) = D_0^{tr}(\omega, \mathbf{k})(\delta_{\alpha\beta} - n_\alpha n_\beta) + D_0^l(\omega, \mathbf{k}) n_\alpha n_\beta, \quad (25)$$

where

$$D_0^{tr}(\omega, \mathbf{k}) = \frac{4\pi}{\varepsilon_0 \omega^2 / c^2 - k^2}, \quad D_0^l(\omega, \mathbf{k}) = \frac{4\pi c^2}{\varepsilon_0 \omega^2}, \quad (26)$$

n_α is the component of the unity vector in the direction of \mathbf{k} , $\delta_{\alpha\beta}$ is the Kroneker's symbol or the unity tensor.

The Green's function of photon field, that includes multiple scattering due to the interaction with thermal fluctuations described by the Hamiltonian (24), can be written as:

$$D_{\alpha\beta}(\omega, \mathbf{k}) = D^{tr}(\omega, \mathbf{k})(\delta_{\alpha\beta} - n_\alpha n_\beta) + D^l(\omega, \mathbf{k}) n_\alpha n_\beta, \quad (27)$$

$$D^{tr}(\omega, \mathbf{k}) = \frac{4\pi}{\varepsilon^{tr} \omega^2 / c^2 - k^2}, \quad D^l(\omega, \mathbf{k}) = \frac{4\pi c^2}{\varepsilon^l \omega^2}, \quad (28)$$

here ε^{tr} and ε^l are, correspondingly, the transverse and the longitudinal components of the dielectric permittivity tensor:

$$\varepsilon_{\alpha\beta}(\omega, \mathbf{k}) = \varepsilon^{tr}(\omega, \mathbf{k})(\delta_{\alpha\beta} - n_\alpha n_\beta) + \varepsilon^l(\omega, \mathbf{k}) n_\alpha n_\beta. \quad (29)$$

It is clear from Eqns (27)-(29) that the problem of finding the dielectric permittivity is equivalent to the problem of finding the Green's function.

Let us calculate the multiple scattering Green's function and the corresponding dielectric permittivity. As a starting zero approximation let us take the Green's function (25) that corresponds to the clear water. The dielectric permittivity of the clear water depends only on the circular frequency ω :

$$\varepsilon_{\alpha\beta}^0(\omega) = \varepsilon_0(\omega)\delta_{\alpha\beta} \equiv \varepsilon_0(\omega)(\delta_{\alpha\beta} - n_\alpha n_\beta) + \varepsilon_0(\omega)n_\alpha n_\beta. \quad (30)$$

It means that Eqn. (30) takes into account only temporal dispersion that is determined by the processes of absorption and emission of photons by the water molecules.

The transverse and longitudinal components of the water dielectric permittivity that takes into account processes of multiple scattering on the hydrosol particles can be expressed as:

$$\varepsilon^{tr}(\omega, \mathbf{k}) = \varepsilon_0(\omega) + \delta\varepsilon^{tr}(\omega, \mathbf{k}), \quad \varepsilon^l(\omega, \mathbf{k}) = \varepsilon_0(\omega) + \delta\varepsilon^l(\omega, \mathbf{k}), \quad \varepsilon_0(\omega) = \varepsilon_0'(\omega) + i\varepsilon_0''(\omega), \quad (31)$$

here ε_0' and ε_0'' are the real and the imaginary parts of the dielectric permittivity of pure water.

To calculate $\delta\varepsilon^{tr}$ and $\delta\varepsilon^l$ let us carry out the standard procedure (Abrikosov, Gorkov and Dzyaloshinski, 1963; Haltrin, 1980) to calculate corrections to the Green's function starting from the Hamiltonian given by Eqn. (24).

The Green's function of the photons in turbid water is expressed through the following Dyson's equation:

$$D_{\alpha\beta}(\omega_n, \mathbf{k}) = D_{\alpha\beta}^{(0)}(\omega_n, \mathbf{k}) + D_{\alpha\gamma}^{(0)}(\omega_n, \mathbf{k})\pi_{\gamma\mu}(\omega_n, \mathbf{k})D_{\mu\beta}(\omega_n, \mathbf{k}), \quad (32)$$

$$\pi_{\alpha\beta}(\omega_n, \mathbf{k}) = -gh \int d^3\mathbf{q} \Delta^2(\mathbf{q}) D_{\alpha\gamma}(\omega_n, \mathbf{k} - \mathbf{q}) \Gamma_{\gamma\beta}(\omega_n, \mathbf{k} - \mathbf{q}, \mathbf{k}), \quad h = \bar{a}^3 \omega_n^4 / (2\pi)^5.$$

where $\pi_{\alpha\beta}$ is the polarization operator, and $\Gamma_{\gamma\beta}$ is the total apex part that corresponds to the sum of all orders of light scattering. The polarization operator is linked with the dielectric permittivity tensor by the equation:

$$\delta\varepsilon_{\alpha\beta}(\omega_n, \mathbf{k}) = (4\pi/\omega_n^2)\pi_{\alpha\beta}(\omega_n, \mathbf{k}). \quad (33)$$

By resolving the Dyson's equation (32) regarding $\Gamma_{\alpha\beta}$, we have the following integral equation for the components of the apex part:

$$\Gamma_{\alpha\beta}(\mathbf{k} - \mathbf{p}, \mathbf{k}) = g\delta_{\alpha\beta} + h \int d^3\mathbf{q} \Delta^2(\mathbf{q}) \Gamma_{\alpha\mu}(\mathbf{k} - \mathbf{p}, \mathbf{k} - \mathbf{p} - \mathbf{q}) \times \quad (34)$$

$$D_{\mu\eta}(\mathbf{k} - \mathbf{p} - \mathbf{q}) \Gamma_{\eta\nu}(\mathbf{k} - \mathbf{p} - \mathbf{q}, \mathbf{k} - \mathbf{q}) D_{\nu\kappa}(\mathbf{k} - \mathbf{q}) \Gamma_{\kappa\beta}(\mathbf{k} - \mathbf{q}, \mathbf{k}).$$

In solving Eqn. (34) let us restrict ourselves to the scattering on large particles, $2\bar{a} \gg \lambda$. In this case $p < q < 1/\bar{a} \ll k$. By representing the apex part as the sum of transverse and longitudinal parts, as in Eqn. (29), integrating over \mathbf{q} , and analytically expanding to the real frequency axis, we obtain the following equations for the components of the apex part:

$$\Gamma^{tr} = g + \frac{\bar{a}^2 \omega^4}{2\pi^2 k^2 c^4} (\Gamma^{tr})^3, \quad \Gamma^l = g + \frac{2}{3\pi^2 (\varepsilon^l)^2} (\Gamma^l)^3. \quad (35)$$

DIELECTRIC PERMITTIVITY

Let us derive equations that link corrections to the dielectric permittivity with the apex part. Using Eqns. (30)-(33), integrating over \mathbf{q} , and analytically transferring to the real frequency axis, we have the following equations:

$$\varepsilon^{rr} = \varepsilon_0 + i g \frac{\bar{a} \omega |\omega|}{16\pi k c^2} \Gamma^{rr}, \quad \varepsilon^l = \varepsilon_0 - \frac{g}{6\pi^2 \varepsilon^l} \Gamma^l. \quad (36)$$

To have a closed system of equations let us add the dispersion relation taken from the Maxwell's equations:

$$k^2 = \varepsilon^{rr} \omega^2 / c^2. \quad (37)$$

Now we have two complete systems of equations for the complex parameters: ε^{rr} and ε^l :

$$\varepsilon^l = \varepsilon_0 - \frac{g}{6\pi^2 \varepsilon^l} \Gamma^l, \quad \Gamma^l = g + \frac{2}{3\pi^2 (\varepsilon^l)^2} (\Gamma^l)^3, \quad (38)$$

$$k^2 = \varepsilon^{rr} \frac{\omega^2}{c^2}, \quad \varepsilon^{rr} = \varepsilon_0 + \frac{i g q}{16\sqrt{\varepsilon^{rr}}} \Gamma^{rr}, \quad \Gamma^{rr} = g + \frac{q^2}{2\varepsilon^{rr}} (\Gamma^{rr})^3, \quad (39)$$

here $q = \bar{a} |\omega| / \pi c$. In the general case of arbitrary values of g (which corresponds to the arbitrary concentrations) the systems of equations (38) and (39) have no analytical solutions. For the value of the parameter

$$g < 4\sqrt{2} \varepsilon'_0 / 3, \quad C_V < 2 \cdot 10^{-3} \quad (\text{or } C_p < 2 \text{ g/m}^3), \quad (40)$$

we have an approximate solution for the longitudinal component of the dielectric permittivity tensor:

$$\varepsilon^l = \varepsilon'_0 \left\{ 1 - \frac{q_C}{3} \sin \left[\frac{1}{3} \sin^{-1} (3q_C) \right] \right\}, \quad q_C = \frac{C_V \delta \varepsilon_h}{\pi \varepsilon'_0} \sqrt{\frac{k_B T}{2\bar{a}^3 \rho u^2}}. \quad (41)$$

Here $C_p \equiv \rho C_V$ is the concentration of hydrosol particles in conventional units (g/m^3).

According to Eqn. (38) ε^l is determined only by the water properties. As it is seen from the Eqns. (38)-(39) this is also true for any water turbidity. For the calculation of the transversal part of the dielectric permittivity let us consider only the case of a weak attenuation of light. Let us write the expression for the absolute value of the photon wave vector:

$$k = k_0 + i(a + b)/2, \quad (42)$$

here k_0 is the real part of the wave vector, a is the absorption coefficient, and b is the scattering coefficient. The condition of weak attenuation $a + b \ll k_0$ is equivalent to the following conditions:

$$|\delta \varepsilon^{rr}| \ll \varepsilon'_0, \quad \text{or } C_V < 2 \cdot 10^{-3} \quad (C_p < 2 \text{ g/m}^3). \quad (43)$$

In this case the solution of Eqns. (39) for the transversal part of the dielectric permittivity is:

$$\delta \varepsilon^{rr} = i \frac{16\pi}{\sqrt{3}} \varepsilon'_0 q_C \text{sign } \omega \begin{cases} 4 \sin \left[\frac{1}{3} \sin^{-1} \zeta \right], & \zeta < 1, \\ \psi_\varepsilon + \frac{1}{\psi_\varepsilon}, & \zeta \geq 1, \end{cases} \quad (44)$$

where

$$\psi_\varepsilon = \sqrt[3]{\zeta - \sqrt{\zeta^2 - 1}}, \quad \zeta = \frac{3C_V \delta \varepsilon_h}{4\pi} \sqrt{\frac{6T}{\bar{a} \rho u^2}} \left(\frac{\omega^2}{k_0 c^2} \right). \quad (45)$$

SCATTERING COEFFICIENT

The light scattering coefficient b is calculated by the substitution of Eqns. (44)-(45) into Eqns. (37), (42). It has the following form:

$$b = \zeta \begin{cases} 4 \sin\left(\frac{1}{3} \sin^{-1}(\zeta \chi)\right), & \zeta \chi < 1, \\ \sqrt[3]{\zeta \chi - \sqrt{\zeta^2 \chi^2 - 1}} + \frac{1}{\sqrt[3]{\zeta \chi - \sqrt{\zeta^2 \chi^2 - 1}}}, & \zeta \chi \geq 1, \end{cases} \quad (46)$$

where

$$\chi = \frac{12 \bar{a} \sqrt{6}}{\pi} \sqrt{\frac{T}{\bar{a}^3 \rho u^2}}, \quad \zeta = C_v \frac{\pi}{8 \lambda} \left(\frac{\delta \epsilon_h}{\epsilon'_0} \right), \quad (47)$$

here λ is the wavelength of light in vacuum.

In the general case of the arbitrary values of the parameter g (or arbitrary concentrations C_v or C_p) the systems of Eqns. (38)-(39) have no solutions in analytic form. However it is easy to solve that system numerically.

At very small concentrations of particles ($C_p < 0.03 \text{ mg/m}^3$) the dependence $b(C_p)$ is quadratic. At the concentrations typical to the open ocean the dependence is linear. At higher concentrations ($5 < C_p < 50 \text{ mg/m}^3$) the dependence reminds the experimental one proposed by Prieur and Sathyendranath (1981).

CONCLUSION

It is shown that the approach based on the Maxwell's theory in a stochastically scattering medium can be productively used in ocean optics. From the solutions of the Dyson's equation the dielectric permittivity tensor of water with the imbedded scattering particles is found. From the equations for the dielectric permittivity a nonlinear dependence of the scattering coefficient of water is derived. It is shown that the scattering coefficient of water nonlinearly depends on the concentration of scatterers. The dependence is quadratic when the concentration of scattering particles is very small. It is linear to the concentration of scatterers at values typical to the open ocean. At the concentrations typical to the coastal waters the concentrational dependence weakens and reaches saturation at values higher than 100 g/m^3 .

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