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Light Scattering Coefficient of Seawater for Arbitrary Concentrations of Hydrosols

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Abstract — The scattering coefficient of seawater as a function of concentration of hydrosol particles is calculated. The approach used is based on the Maxwell's equations in a stochastically scattering seawater. The water is modeled as thermally fluctuated medium filled with the hydrosol particles. It is found that the scattering coefficient quadratically depends on concentration when the concentration of scatterers is very small. The scattering coefficient is linear to concentration of scatterers at values typical to the open ocean. At the values of concentrations typical to coastal waters the dependence on concentration weakens and reaches the saturation at very high values.

INTRODUCTION

In a majority of publications available today the light scattering coefficient by seawater is considered as linearly dependent on concentrations of hydrosols. The experiments by Prieur and Sathyendranath [1] show that at certain concentrations of chlorophyll C_C, typical to coastal waters, the dependence of seawater absorption coefficient on C_C is nonlinear. Clark and Backer [2] showed that the scattering coefficient of seawater is non-linearly dependent on chlorophyll concentration that is strongly correlated to the concentration of scattering matter of biologic origin.

In this paper an attempt is made to develop an approach to calculate the scattering coefficient of seawater as a function of concentration of hydrosol particles C_p. The approach is based on the solutions of the Maxwell's equations in a stochastically scattering medium (seawater). The seawater is modeled as thermally fluctuated medium filled with the hydrosol scatterers.

The final result of this paper is Eqn. (22) for the seawater scattering coefficient. The scattering coefficient b is linear with concentration of scattering particles at values typical to the open ocean. The coefficient quadratically depends on concentration when the concentration is very small (typical for the Sargasso Sea waters). At the concentrations close to the values that are typical to coastal waters, the dependence on concentrations weakens and reaches saturation at very high values. The results of this paper can explain some experimental data obtained in turbid coastal waters. They also can give a reasonable explanation to the phenomenon that some very clean ocean waters seem to be more transparent than the distilled water.

APPROACH

The radiative transfer theory, predominantly used in ocean optics, cannot explain nonlinear dependence of inherent optical properties on concentration of scatterers. In order to investigate this problem we should start from the Maxwell's equations in stochastically scattering medium. The mathematical formalism of scattering in stochastic medium is identical to the formalism of quantum statistical mechanics [3]. The photons themselves are always quantum particles. For these reasons we have chosen to use the quantum-mechanical statistical approach formulated in Ref. [4].

In this paper the scattering coefficient on hydrosol particles is calculated through the dielectric permittivity of the hydrosol component of the water. The dielectric permittivity (DP) is a constituent part of the Fourier transform of the Green's function of the Maxwell's equations in seawater. By definition, the Green's function is a solution of these equations when a source function is assumed as an infinitely short and localized at one point light pulse [5]. The approach adopted here is based on the theory proposed by the author in Refs. [6, 7] but abandoned at the time due to the lack of interest in coastal hydrooptics.

Hydrosol particles in this model are perceived as small potential holes in a Brownian motion. They are characterized by the size distribution function: $\varphi(a)$ $(\int_0^\infty \varphi(a)\,da = 1)$. Such a model allows one to derive the following interaction Hamiltonian between photons and thermal density fluctuations [6]:

$$\hat{H}_{\text{int}}(t) = -\frac{g}{c^2}\int d^3r\,\frac{\partial \hat{A}_\alpha(\mathbf{r},t)}{\partial t}\,\hat{\psi}(\mathbf{r},t)\,\frac{\partial \hat{A}_\alpha(\mathbf{r},t)}{\partial t}, \quad \alpha = 1,2,3, \tag{1}$$

here $\hat{A}_\alpha$ is a photon field operator which corresponds to a vector potential of light wave, $\mathbf{r}$ is a coordinate, t is time, index α denotes vector's component, c is the speed of light,

$$g = C_V\,\delta\varepsilon_H\sqrt{\frac{k_B T}{\bar{a}^3 \rho u^2}} \sim \gamma\, C_V, \quad \gamma \approx 10^3 \div 10^5, \tag{2}$$

C_V is the volume concentration of hydrosol, $\delta\varepsilon_H$ is the difference between average DP of the hydrosol and DP of pure water, k_B is the Boltzmann's constant, ρ is the water density, and u is the average velocity of a Brownian movement in water. Repeating indices everywhere in this article imply summation. The operator of thermal density fluctuations $\hat{\psi}$ is determined by the following equation [6]:

$$\hat{\psi}(\mathbf{r},t) = \frac{1}{4\pi}\sqrt{\bar{a}^3/(k_B T)}\int \Delta(\mathbf{r}-\mathbf{r}')\,\Phi(\mathbf{r}',t)\,d\mathbf{r}',$$
$$\Delta(\mathbf{r}) = \int_r^\infty \varphi(a)\,da \Big/ [4\pi\int_0^\infty r^2 dr \int_r^\infty \varphi(a)\,da], \tag{3}$$

here $\bar{a} = \int_0^\infty \varphi(a)\,a\,da$ is a mean radius of hydrosol particles, T is the absolute temperature.

THE GREEN'S FUNCTION

Let us write a Green's function of photons propagating in a nonscattering medium with the dielectric permittivity ε_0. It can be represented as a sum of the transverse and longitidinal components [6]. In the energetic-momentum representation the Green's function has the following form:

$$D^{(0)}_{\alpha\beta}(\omega,\mathbf{k}) = D_0^{tr}(\omega,\mathbf{k})(\delta_{\alpha\beta} - n_\alpha n_\beta) + D_0^l(\omega,\mathbf{k})\, n_\alpha n_\beta, \quad (4)$$

where

$$D_0^{tr}(\omega,\mathbf{k}) = \frac{4\pi}{\varepsilon_0\,\omega^2/c^2 - k^2},\quad D_0^l(\omega,\mathbf{k}) = \frac{4\pi\,c^2}{\varepsilon_0\,\omega^2}, \quad (5)$$

n_α is the component of the unity vector in the direction of $\mathbf{k}$, $\delta_{\alpha\beta}$ is the Kroneker's symbol or the unity tensor.

The Green's function of photon field, that includes multiple scattering due to the interaction with thermal fluctuations described by the Hamiltonian (1), can be written as:

$$D_{\alpha\beta}(\omega,\mathbf{k}) = D^{tr}(\omega,\mathbf{k})(\delta_{\alpha\beta} - n_\alpha n_\beta) + D^l(\omega,\mathbf{k})\, n_\alpha n_\beta, \quad (6)$$

$$D^{tr}(\omega,\mathbf{k}) = \frac{4\pi}{\varepsilon^{tr}\,\omega^2/c^2 - k^2},\quad D^l(\omega,\mathbf{k}) = \frac{4\pi\,c^2}{\varepsilon^l\,\omega^2}, \quad (7)$$

here ε^{tr} and ε^l are, correspondingly, the transverse and the longitidinal components of the following dielectric permittivity tensor:

$$\varepsilon_{\alpha\beta}(\omega,\mathbf{k}) = \varepsilon^{tr}(\omega,\mathbf{k})(\delta_{\alpha\beta} - n_\alpha n_\beta) + \varepsilon^l(\omega,\mathbf{k})\, n_\alpha n_\beta. \quad (8)$$

It is clear from Eqns (6)-(8) that the problem of finding the dielectric permittivity is equivalent to the problem of finding the Green's function.

Let us calculate the multiple scattering Green's function and the corresponding dielectric permittivity. As a starting zero approximation let us take the Green's function (4) which corresponds to the clear water. The DP of the clear water depends only on the circular frequency ω:

$$\varepsilon^0_{\alpha\beta}(\omega) = \varepsilon_0(\omega)\,\delta_{\alpha\beta} \equiv \varepsilon_0(\omega)(\delta_{\alpha\beta} - n_\alpha n_\beta) + \varepsilon_0(\omega) n_\alpha n_\beta. \quad (9)$$

It means that Eqn. (9) takes into account only temporal dispersion that is determined by the processes of absorption and emission of photons by the water molecules.

The transverse and longitidinal components of the seawater DP that take into account processes of multiple scattering on the hydrosol particles can be expressed as:

$$\begin{aligned}\varepsilon^{tr}(\omega,\mathbf{k}) &= \varepsilon_0(\omega) + \delta\,\varepsilon^{tr}(\omega,\mathbf{k}),\\ \varepsilon^{l}(\omega,\mathbf{k}) &= \varepsilon_0(\omega) + \delta\,\varepsilon^{l}(\omega,\mathbf{k}),\\ \varepsilon_0(\omega) &= \varepsilon_0'(\omega) + i\,\varepsilon_0''(\omega).\end{aligned} \quad (10)$$

here ε_0' and ε_0'' are the real and the imaginary parts of the dielectric permittivity of pure water.

In order to calculate $\delta\,\varepsilon^{tr}$ and $\delta\,\varepsilon^l$ let us carry out the standard procedure [4, 6] to calculate corrections to the Green's function starting from the Hamiltonian given by Eqn. (1).

The Green's function of the photons in turbid water is expressed through the following Dyson's equation:

$$\begin{aligned}D_{\alpha\beta}(\omega_n,\mathbf{k}) &= D^{(0)}_{\alpha\beta}(\omega_n,\mathbf{k}) + D^{(0)}_{\alpha\gamma}(\omega_n,\mathbf{k})\,\pi_{\gamma\mu}(\omega_n,\mathbf{k})\,D_{\mu\beta}(\omega_n,\mathbf{k}),\\ \pi_{\alpha\beta}(\omega_n,\mathbf{k}) &= -gh\int d^3\mathbf{q}\,\Delta^2(\mathbf{q})\,D_{\alpha\gamma}(\omega_n,\mathbf{k}-\mathbf{q})\,\Gamma_{\gamma\beta}(\omega_n,\mathbf{k}-\mathbf{q},\mathbf{k}),\\ h &= \bar{a}^3\omega_n^4/(2\pi)^5,\end{aligned} \quad (11)$$

where $\pi_{\alpha\beta}$ is the polarization operator, and $\Gamma_{\gamma\beta}$ is the total apex part that corresponds to the sum of all orders of light scattering. The polarization operator is linked with the dielectric permittivity tensor by the equation:

$$\delta\,\varepsilon_{\alpha\beta}(\omega_n,\mathbf{k}) = \left(4\pi/\omega_n^2\right)\pi_{\alpha\beta}(\omega_n,\mathbf{k}). \quad (12)$$

By resolving the Dyson's equation (11) in respect to $\Gamma_{\alpha\beta}$, we have the following integral equation for the components of the apex part:

$$\begin{aligned}\Gamma_{\alpha\beta}(\mathbf{k}-\mathbf{p},\mathbf{k}) &= g\,\delta_{\alpha\beta} + h\int d^3\mathbf{q}\,\Delta^2(\mathbf{q})\,\Gamma_{\alpha\mu}(\mathbf{k}-\mathbf{p},\mathbf{k}-\mathbf{p}-\mathbf{q})\times \quad (13)\\ &D_{\mu\eta}(\mathbf{k}-\mathbf{p}-\mathbf{q})\,\Gamma_{\eta\nu}(\mathbf{k}-\mathbf{p}-\mathbf{q},\mathbf{k}-\mathbf{q})\,D_{\nu\kappa}(\mathbf{k}-\mathbf{q})\,\Gamma_{\kappa\beta}(\mathbf{k}-\mathbf{q},\mathbf{k}).\end{aligned}$$

In solving Eqn. (13) let us restrict ourselves to the scattering on hydrosols with the large average size of particles, $2\,\bar{a} >> \lambda$. In this case $p < q < 1/\bar{a} << k$. By representing the apex part as the sum of transverse and longitudinal parts similar to Eqn. (8), integrating over $\mathbf{q}$, and analytically expanding to the real frequency axis, we obtain the following equations for the components of the apex part:

$$\Gamma^{tr} = g + \frac{\bar{a}^2\omega^4}{2\pi^2 k^2 c^4}\left(\Gamma^{tr}\right)^3,\quad \Gamma^l = g + \frac{2}{3\pi^2(\varepsilon^l)^2}\left(\Gamma^l\right)^3. \quad (14)$$

THE DIELECTRIC PERMITTIVITY

Let us derive equations that link corrections to the dielectric permittivity with the apex part. Using Eqns. (9)-(12), integrating over $\mathbf{q}$, and analytically transferring to the real frequency axis, we have the following equations:

$$\varepsilon^{tr} = \varepsilon_0 + i\,g\frac{\bar{a}\,\omega|\omega|}{16\pi\,k\,c^2}\Gamma^{tr},\quad \varepsilon^l = \varepsilon_0 - \frac{g}{6\pi^2\varepsilon^l}\Gamma^l. \quad (15)$$

In order to have a complete system of equations let us add the dispersion relation taken from the Maxwell's equations:

$$k^2 = \varepsilon^{tr}\omega^2/c^2. \quad (16)$$

Now we have the complete system of five equations (14)-(16) for the five complex parameters: k, Γ^{tr}, Γ^l, ε^{tr}, and ε^l. In the general case of arbitrary values of g (which correspond to the arbitrary concentrations) this system of equations has no analytical solutions. For the value of the parameter

$$g < 4\sqrt{2}\,\varepsilon_0'/3,\quad C_V < 2\cdot 10^{-3}\ \left(\text{or}\ \ C_p < 2\ \ g/m^3\right), \quad (16)$$

we have the approximate solution for the longitudinal component of the dielectric permittivity tensor: (17)

$$\varepsilon^l = \varepsilon_0'\left\{1 - \frac{q_C}{3}\sin\left[\frac{1}{3}\sin^{-1}\left(3\,q_C\right)\right]\right\},\quad q_C = \frac{C_V\delta\varepsilon_H}{\pi\,\varepsilon_0'}\sqrt{\frac{k_B\,T}{2\,\bar{a}^3\rho\,u^2}}.$$

Here $C_p \equiv \rho\,C_V$ is the concentration of hydrosol particles in conventional units (g/m^3).

According to Eqn. (17) ε^l is determined only by the water properties. As it is seen from the Eqns. (14)-(16) this is also true for any water turbidity. For the calculation of the transversal part of the dielectric permittivity let us consider only the case of a weak attenuation of light. Let us write the expression for the absolute value of the photon wave vector:

$$k = k_0 + i\,(a+b)/2, \quad (18)$$

here k_0 is the real part of the wave vector, a is the absorption coefficient, and b is the scattering coefficient. The condition of weak attenuation $a + b << k_0$ is equivalent to the following conditions:

$$\left|\delta\,\varepsilon^{tr}\right| << \varepsilon_0',\ \text{or}\ \ C_V < 2\cdot 10^{-3}\ \left(C_p < 2\ \ g/m^3\right). \quad (19)$$

In this case the solution of Eqns. (14)-(16) for the transversal part of the dielectric permittivity is:

$$\delta\varepsilon^{tr} = i\frac{16\pi}{\sqrt{3}}\varepsilon_0' q_C \,\mathrm{sign}\,\omega \begin{cases} 4\sin\left[(1/3)\sin^{-1}\zeta\right], \zeta<1, \\ \psi_\varepsilon + 1/\psi_\varepsilon, \quad \zeta \ge 1, \end{cases} \quad (20)$$

where
$$\zeta = \frac{3C_V\delta\varepsilon_H}{4\pi}\sqrt{\frac{6k_BT}{\bar{a}\rho u^2}}\left(\frac{\omega^2}{k_0c^2}\right), \psi_\varepsilon = \sqrt[3]{f-\sqrt{f^2-1}}. \quad (21)$$

SCATTERING COEFFICIENT

The light scattering coefficient b is calculated by the substitution of Eqn. (20) into Eqns. (16) and (18). It has the following form:

$$b = \frac{\pi C_V\delta\varepsilon_H}{8\varepsilon_0'\lambda} \begin{cases} 4\sin\left(\frac{1}{3}\sin^{-1}\xi\right), \xi<1, \\ \psi_b + 1/\psi_b, \quad \xi\ge 1, \end{cases} \quad (22)$$

where
$$\xi = \frac{3C_V\delta\varepsilon_H}{2\varepsilon_0'\lambda}\sqrt{\frac{6k_BT}{\bar{a}\rho u^2}}, \quad \psi_b = \sqrt[3]{\xi-\sqrt{\xi^2-1}}, \quad (23)$$

here λ is the wavelength of light in vacuum.
If $\xi \ll 1$ Eqns. (22)-(23) can be further simplified:

$$b = \frac{\pi^2 C_V^2\delta\varepsilon_H^2 k_BT}{4\varepsilon_0'^2\rho\bar{a}^2u^2\lambda^2}, \quad C_V < 10^{-5}\left(\text{or } C_p < 10^{-2}\ g/m^3\right). \quad (24)$$

Equation (23) coincides with the formula for the scattering coefficient derived in Ref. [6] for the case of $\lambda \ll 2\bar{a}$.

In the general case of the arbitrary values of the parameter g (or arbitrary concentrations C_V or C_p) the system of Eqns. (14)-(16) has no solutions in analytic form. However it is easy to solve that system numerically. Figure 1 shows the dependence of the scattering coefficient of seawater as a function of the hydrosol particles concentration. The absolute values of b are not shown because they depend on the average radius of scattering particles $\bar{a}$ and the average density of the particles ρ that vary from a region to a region. The idea of displaying this picture is to show the dependence of the scattering coefficient b on concentration. At very small concentrations of particles ($C_p < 0.03\,mg/m^3$) the dependence $b(C_p)$ is quadratic. At the concentrations typical to the open ocean the dependence is linear. At higher concentrations ($5 < C_p < 50\,mg/m^3$) the dependence shown in Fig. 1 reminds the exprerimental one proposed in Ref. [1]. At even larger concentrations ($C_p > 50\ mg/m^3$) it reaches saturation.

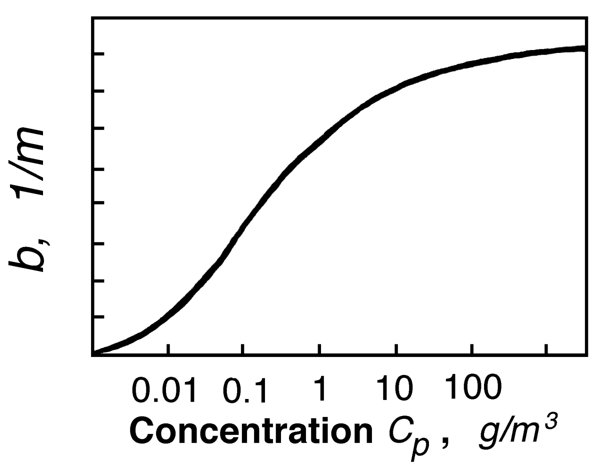

Figure 1. The scattering coefficient of seawater filled with the hydrosol particles as a function of the hydrosol particles concentration.

CONCLUSION

It is shown that the approach based on the Maxwell's theory in a stochastically scattering medium can be productively used in ocean optics. From the solutions of the Dyson's equation the dielectric permittivity tensor of seawater with the scattering particles is found. From the equations for the dielectric permittivity a nonlinear dependence of the scattering coefficient of water is derived. It is shown that the scattering coefficient of seawater quadratically depends on the concentration when the concentration of scattering particles is very small. It is linear to the concentration of scatterers at values typical to the open ocean. At the concentrations typical to the coastal waters the concentrational dependence weakens and reaches saturation at values higher than $100\,g/m^3$.

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