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Fresnel reflection coefficient of very turbid waters*

Vladimir I. Haltrin

Naval Research Laboratory, Ocean Sciences Branch, Code 7331
Stennis Space Center, MS 39529-5004, USA. e-mail: <haltrin@nrlssc.navy.mil>
Telephone: 228-688-4528; fax: 228-688-5379

ABSTRACT

Results of an investigation of the dependence of the Fresnel reflection coefficient for marine and lake waters on concentrations of dissolved organic matter are presented.

The numerical computations of the Fresnel reflection coefficient as a function of incidence angle, wavelength of light and chlorophyll concentration show that starting from concentrations of chlorophyll near 120 mg/m^3 and increasing to 160 mg/m^3 the Fresnel reflection coefficient at all incidence angles grows significantly. It reaches 100% at a concentration of chlorophyll approximately 170 mg/m^3. At a concentration near 140 mg/m^3 the ultra-eutrophic water reflects all light except the red component. At chlorophyll concentrations larger than 170 mg/m^3 the ultra-eutrophic water is optically equivalent to a perfectly reflecting mirror.

1.0 INTRODUCTION

The Fresnel reflection coefficient of light is a function of an angle of incidence and water refraction coefficient. The water refraction coefficient is a complex value with the imaginary part of the order of $1 \cdot 10^{-7}$ to $1 \cdot 10^{-4}$ for the cases of waters where the chlorophyll concentration is less than 12 mg/m^3. Such small values of the imaginary part of the refractive index have no influence on the Fresnel reflection coefficient. With the increase of water turbidity the imaginary part of refractive index approaches a value of ten. In this case the concentrational dependence of the Fresnel reflection coefficient becomes significant.

The imaginary part of the water refractive index is proportional to the microscopic attenuation coefficient of water in a layer with a thickness of one wavelength of light. The microscopic attenuation coefficient does not include scattering by large particles and absorption by chlorophyll *in-vivo*. The imaginary part of the refractive index is proportional to the sum of the pure water attenuation coefficient and absorption coefficient by dissolved organic matter, that includes yellow substance and chlorophyll found in water outside phytoplankton cells.

2.0 FRESNEL REFLECTION COEFFICIENT OF LIGHT FROM A FLAT SURFACE OF A MEDIUM WITH COMPLEX REFRACTIVE INDEX

The Fresnel reflection coefficient of non-polarized light is determined by the following equation:

$$R_F(\varphi,\psi) = \frac{1}{2}\left(R_F^p + R_F^t\right), \qquad (1)$$

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here R_F^p and R_F^t are, respectively, Fresnel reflection coefficients of parallel and transverse polarized light. They are expressed by the following equations:

$$R_F^p = \left|\frac{n_w \cos\varphi - \cos\psi}{n_w \cos\varphi + \cos\psi}\right|^2, \quad R_F^t = \left|\frac{\cos\varphi - n_w \cos\psi}{\cos\varphi + n_w \cos\psi}\right|^2, \tag{2}$$

where the angle ψ is given by the relationship derived from Snell's law:

$$\cos\psi = \frac{e^{i\psi} + e^{-i\psi}}{2} = \sqrt{1 - \frac{\sin^2\varphi}{n_w^2}}. \tag{3}$$

here φ is a striking angle, and ψ is an angle of refraction. The less known feature of Fresnel equations (1)-(3) (see, for example, Sivukhin, 1985) is that these equations are valid for complex values of refractive index $n_w = n'_w + i\,n''_w$. In this case the value $\psi = \psi' + i\,\psi''$ is complex and the angle of refraction is determined by its real part ψ'.

In spite of the apparent simplicity of equations (1)-(3) it is extremely difficult to analytically express Fresnel reflection coefficients R_F^p and R_F^t through the real values φ, n'_w and n''_w. Fortunately, numerical calculations of these Fresnel reflection coefficients are very easy (see APPENDIX A with a FORTRAN function to calculate R_F, R_F^p and R_F^t).

Figure 1 shows an example of calculations of these three Fresnel reflection coefficients for a water with an imaginary part of refraction index varying between 0 and 10.

3.0 CONNECTION BETWEEN IMAGINARY PART OF REFRACTIVE INDEX AND WATER ABSORPTION COEFFICIENT

The amplitude of electric or magnetic field ($\mathbf{E}$ or $\mathbf{H}$) attenuates according to the law:

$$\mathbf{E},\mathbf{H} \sim e^{i\omega t - ikz}, \quad k = \frac{\omega}{c} n_w \equiv \frac{\omega}{c}\left(n'_w - i\,n''_w\right), \tag{4}$$

here k is a wave vector, ω is a circular frequency of light, c is a vacuum speed of light, and n_w

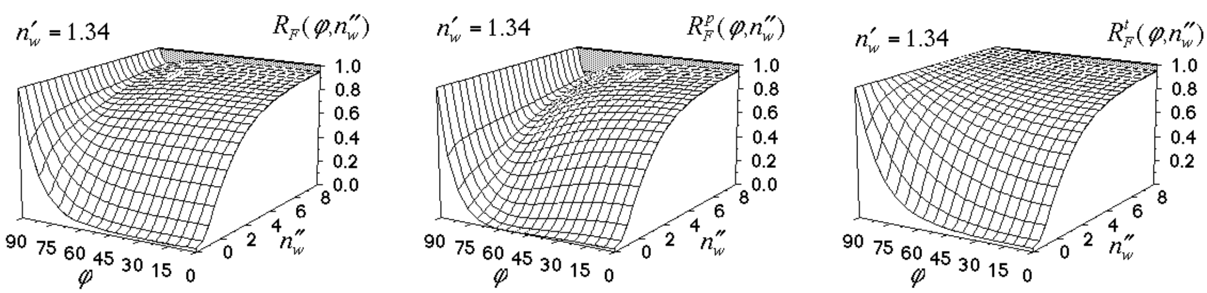

Figure 1. Fresnel reflection coefficients of water R_F with the imaginary part of refractive index varying between 0 and 10.

$(n_w = n'_w - i\,n''_w)$ is a complex refraction index of water, *z* is a spatial coordinate.

According to electrodynamics theory the radiance of light is attenuated as follows:

$$L \sim \mathbf{EE}^* \sim \exp(-2\omega\, n''_w\, z / c). \tag{5}$$

At the same time according to hydrooptics the radiance of light in a thin subsurface water layer, comparable with light wavelength $(\sim \lambda = 2\pi c / \omega)$, where we can neglect scattering processes, is equal:

$$L \sim \exp(-a_{diss} z), \tag{6}$$

here a_{diss} is an absorption by pure water and all dissolved absorbing substances.

Comparing Eqs. (5) and (6) we have the following equation that relates the imaginary part of refractive index n''_w to the dissolved part of the absorption coefficient, a_{diss}:

$$n''_w = \frac{a_{diss}\lambda}{4\pi}. \tag{7}$$

The values of a_{diss} and λ in this equation should be measured in the same units (*m*, for example).

Consequently, the complex refraction index of water can be expressed as:

$$n_w = n'_w - i\,\frac{a_{diss}\,\lambda}{4\pi}, \quad n'_w \cong 1.34. \tag{8}$$

4.0 MODEL OF WATER ABSORPTION COEFFICIENT

Now we define a_{diss} and use Eq. (8) with Fresnel equations (1)-(3) in order to model specular reflection from a surface of a very turbid water.

In order to calculate an imaginary part of refractive index we need to know a part of an absorption coefficient of water that influence effects of light reflection and refraction in the vicinity of air-water interface. We start from a one-parameter model that connects absorption and backscattering coefficients a and b_B of water with the concentration of chlorophyll C_c (Haltrin, 1999). This model is tuned to reproduce the same color indices that are used for remote restoration of chlorophyll content from satellite measurements.

The total absorption coefficient of water $a(\lambda)$, (m^{-1}) is taken to be (Prieur, and Sathyendranath, 1981; Haltrin, 1985; Hawes, Carder, and Harvey, 1992):

$$\left.\begin{aligned} a(\lambda) = \quad & a_w(\lambda) + a_c(\lambda) + a_f(\lambda) + a_h(\lambda), \quad && a_f(\lambda) = 35.959\, C_f \exp(-0.0189\,\lambda), \\ & a_c(\lambda) = a_c^0(\lambda)\, C_c^{\,0.602}, && a_h(\lambda) = 18.828\, C_h \exp(-0.01105\,\lambda), \end{aligned}\right\}, \tag{9}$$

where $a_w(\lambda)$ is the pure water absorption coefficient in m^{-1}, λ is the vacuum wavelength of light in nm, $a_0^c(\lambda)$ is the specific absorption coefficient of chlorophyll in m^{-1}, C_c is the total concentration of chlorophyll in mg/m^3, C_f and C_h are, respectively, the concentrations of fulvic and humic acids in mg/m^3.

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Let us estimate the portion of an absorption coefficient that influences water refractive index. The pure water absorption coefficient $a_w(\lambda)$ may be neglected because its influence on a Fresnel reflection coefficient is negligent. The predominant part of chlorophyll is located in phytoplankton cells and may be disregarded too. We only take into account a portion of chlorophyll $0 \le p_{diss} \le 1$ that is actually leaked into water. There is no unanimous agreement among researchers concerning the actual value of p_{diss}. Some investigators believe that there is no dissolved chlorophyll in water. Others suggest that there is some leakage of live chlorophyll from phytoplankton cells. This leakage is caused by zooplankton grazing and natural deterioration, and the portion p_{diss} depends on the age of phytoplankton community, amount of zooplankton, and processes of natural conversion of phytoplankton into yellow substance. In our computations we adopt conservative p_{diss}=0.1. It should be noted that the yellow substance is the major factor in an anomalous behavior of the Fresnel reflection coefficient and our results will change only slightly if we accept that p_{diss}=0.

Finally, we accept the following equations for the dissolved part of absorption coefficient:

$$a_{diss}(\lambda) \cong p_{diss} a_c(\lambda) + a_f(\lambda) + a_h(\lambda), \quad p_{diss} \approx 0.1, \tag{10}$$

The concentrations C_c, C_h, and C_f, of dissolved matter are connected with the chlorophyll concentration C_c as follows (Haltrin, 1999):

$$C_f = 0.782 \cdot C_c \cdot \exp\left(0.800 + 0.123 \cdot C_c \right), \quad C_h = 0.337 \cdot C_c \cdot \exp\left(-0.554 + 0.123 \cdot C_c \right). \tag{11}$$

By combining Eqs. (8)-(11) we have the following formula for the complex refraction coefficient of water n_w that expresses n_w only through two parameters: wavelength of light λ in *nm* and chlorophyll concentration C_c in mg/m^3,

$$\begin{aligned} n_w = 1.34 - i\lambda \big[& 2.2377 \cdot 10^{-9} \cdot C_c \cdot \exp\left(0.800 + 0.123 \cdot C_c - 0.0189\lambda \right) \\ & + 5.049 \cdot 10^{-10} \cdot C_c \cdot \exp\left(-0.554 + 0.123 \cdot C_c - 0.01105\lambda \right) + 7.96 \cdot 10^{-11} p_{diss} \, a_c^0(\lambda) \, C_c^{\,0.602} \big], \end{aligned} \tag{12}$$

here p_{diss}=0.1, and the values for chlorophyll specific absorption coefficient $a_c^0(\lambda)$ should be taken from the paper by Prieur and Sathyendranath (1981).

5.0 MODELING OF FRESNEL REFLECTION COEFFICIENT OF VERY TURBID WATERS

The code used to calculate Fresnel reflection coefficient n_w of light from a surface of a very turbid water is given in APPENDIX B. Figure 2 shows spectral and angular dependence of n_w for six concentration of chlorophyll: C_c = 110, 120, 130, 140, 150 and 160 mg/m^3.

The results of computations show that starting from concentrations of chlorophyll near 120 mg/m^3 and increasing to 160 mg/m^3 the Fresnel reflection coefficient at all incidence angles grows significantly. It reaches 100% at a concentration of chlorophyll approximately 170 mg/m^3. At a concentrations approaching 140 mg/m^3 the ultra-eutrophic water reflects all light except the red component. At chlorophyll concentrations larger than 170 mg/m^3 the ultra-eutrophic water is optically equivalent to a perfectly reflecting mirror.

6.0 CONCLUSIONS

The imaginary part of the water refractive index is proportional to the microscopic attenuation coefficient of water in a layer with a thickness of one wavelength of light. The microscopic attenuation coefficient does not include scattering by large particles and absorption by chlorophyll *in-vivo*. The imaginary part of the refractive index is proportional to the sum of the pure water attenuation coefficient and absorption coefficient by dissolved organic matter, which includes yellow substance and chlorophyll found in water outside phytoplankton cells. At concentrations of chlorophyll C_c below 110 mg/m^3 the Fresnel reflection coefficient R_F is not sensitive to the chlorophyll concentration and practically coincides with the value typical to pure water. When C_c varies between 110 and 160 mg/m^3 values of R_F undergo rapid changes (see Fig. 2). At C_c higher than 170 mg/m^3 the water reflects almost all visible light.

7.0 ACKNOWLEDGMENTS

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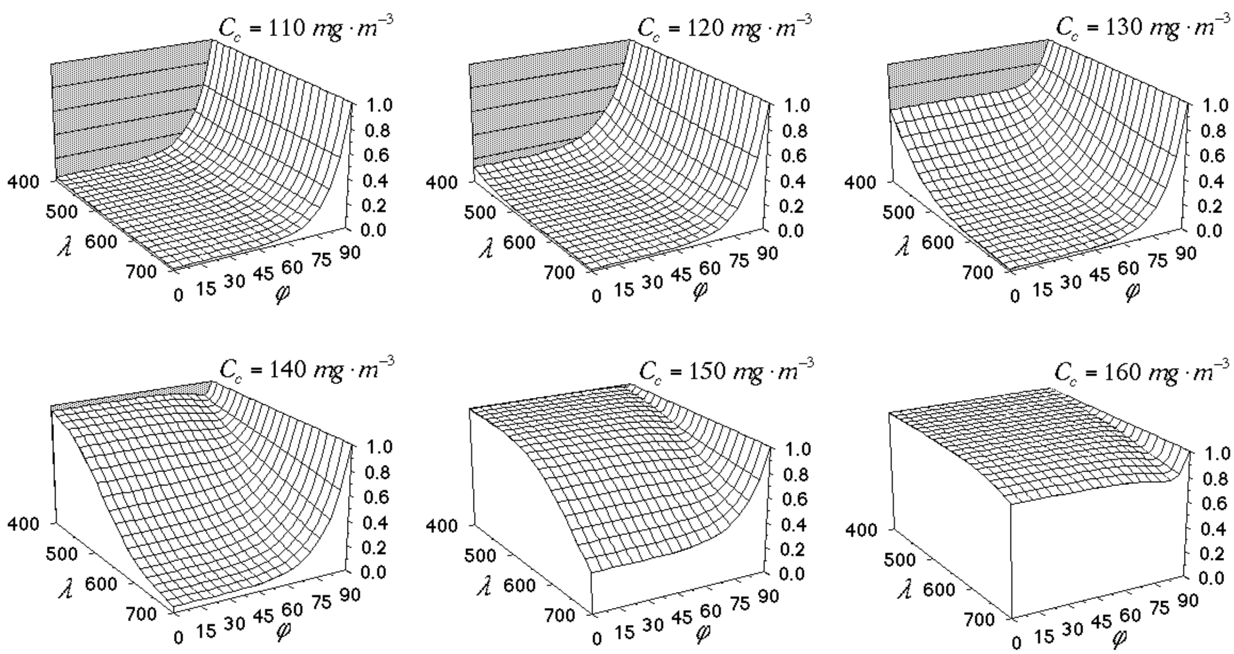

Figure 2. Fresnel reflection coefficient R_F of light from a surface of water with very high concentrations of chlorophyll.

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APPENDIX A. FORTRAN FUNCTION TO COMPUTE FRESNEL REFLECTION COEFFICIENT OF LIGHT

```
! *********************************************************************
      extended function Rfresnel(nRef,fi, rfp,rft)
! *********************************************************************
!  input: nRef -  relative complex refractive index of the medium
!         fi -    light striking angle (in degrees)
!  output: Rfresnel - Fresnel Reflection coefficient for unpolarized light
!        rfp - Fresnel Reflection coefficient for parallel polarized light
!      rfp - Fresnel Reflection coefficient for transverse polarized light
! *********************************************************************
      implicit     none
      extended     rfp,rft,fi,sinFi,cosFi
      complex*16   nRef,sinPsi,cosPsi,r

        sinFi = SIND(fi)
        cosFi = SQRT(1.-sinFi*sinFi)
        sinPsi = sinFi/nRef
        cosPsi = CSQRT(1.-sinPsi*sinPsi)

          r = nRef*cosPsi
          r = (cosFi-r)/(cosFi+r)
        rft = ABS(r)
        rft = rft*rft

          r = nRef*cosFi
          r = (r-cosPsi)/(r+cosPsi)
        rfp = ABS(r)
        rfp = rfp*rfp

      Rfresnel = 0.5*(rft + rfp)

      return
      end
! *********************************************************************
```

APPENDIX B. FORTRAN FUNCTION TO COMPUTE COMPLEX REFRACTION COEFFICIENT OF WATER

```
! *********************************************************************
!     This function calculates water refraction coefficient
! =====================================================================
!  input:    lam - wavelength of light in nm (400 ≤ lam ≤ 700)
!            Cc -  chlorophyll concentration in mg/m^3 (Cc > 0.)
! =====================================================================
!   parameter pdiff determines the share of chlorophyl dissolved in water
!   parameter rr is a real part of refractive index
! *********************************************************************
      complex*16 function fnWater(lam,Cc)
! *********************************************************************
      implicit   none
      integer*2  i
      extended   lam,Cc,rr,ri,s,q,pdiff,ac0(61)
```

```
      data       rr /1.34/, pdiff  /0.1/
      data       ac0 /0.687,0.781,0.828,0.883,0.913,0.939,0.973,1.001
     &               ,1.000,0.971,0.944,0.928,0.917,0.902,0.870,0.839
     &         ,0.798,0.773,0.750,0.717,0.668,0.645,0.618,0.582,0.528
     &         ,0.504,0.474,0.444,0.416,0.384,0.357,0.321,0.294,0.273
     &         ,0.276,0.268,0.291,0.274,0.282,0.249,0.236,0.279,0.252
     &         ,0.268,0.276,0.299,0.317,0.333,0.334,0.326,0.356,0.389
     &         ,0.441,0.534,0.595,0.544,0.502,0.420,0.329,0.262,0.215/

        s = 0.2*(lam-395.)
        i = INT(s)
        s = s - i
        q = 0.06*(ac0(i)*(1.-s) + ac0(i+1)*s)
        s = (2.2377e-9)*EXP(0.8+0.123*Cc-0.0189*lam)
        s = s + (5.049e-10)*EXP(-0.554+0.123*Cc-0.011*lam)
        s = s*Cc + (7.96e-11)*pdiff *q*(Cc**0.602)
        ri = -lam*s
      fnWater = CMPLX(rr,ri)

      return
      end
! **********************************************************************
```

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