Matrix Equation for Radiative Transport in Sea-Atmospheric System

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ABSTRACT

The proposed approach is based on the theory developed for the marine environment. This approach is tuned to be valid in the whole range of optical properties including an unrealistic case of a totally scattering sea. Because the atmosphere below the ozone layer is totally scattering in the visible range of spectrum, the proposed approach may be applied to the atmosphere by setting the absorption coefficient to zero. The solutions to the light scattering problem for oceanic and atmospheric parts are obtained. The one-parameter optical models of seawater and atmosphere are presented. The proposed model allows to calculate spectral albedo of the ocean-atmosphere system as a function of three input parameters: the chlorophyll concentration, the aerosol atmospheric optical thickness at near infrared, and the solar zenith angle.

Keywords: radiative transfer, scattering, ocean optics, atmospheric optics

1. INTRODUCTION

Optical properties of the sea and the clear atmosphere are very different. Unclouded atmosphere without fog is always optically thin in the vertical direction. The sea, with the exception of some rare cases of clean and very shallow waters, is always optically thick. This difference usually results in implementing different approaches to calculate light fields in these environments. Real-time atmospheric correction algorithms may be improved with respect to the speed of calculation, if the underlying theories for atmospheric and oceanic light propagation are unified. This work presents a unified approach to light propagation in ocean-atmospheric environment that may be useful for some remote sensing applications.

2. EQUATIONS FOR IRRADIANCES IN THE OCEAN

Let us consider a homogeneous scattering and absorbing ocean illuminated by the light of Sun and sky. The system of equations for downward \( E_1 \) and upward \( E_2 \) irradiances of scattered light is written as follows:\(^1\text{-}^3\)

\[
\begin{align*}
\frac{d}{dz} + (2 - \mu_t)(a + b_t) & E_1(z) - (2 + \mu_t)b_t E_1(z) = f_1(z), \\
-(2 - \mu_t)b_t E_1(z) + \frac{d}{dz} + (2 + \mu_t)(a + b_t) & E_2(z) = f_2(z).
\end{align*}
\]

\(1\)

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The cartesian coordinate system here is chosen in such a way that the 0z axis is directed from the sea surface to the bottom. Here $a$ is an absorption coefficient, $b_B$ is a backscattering coefficient, $\bar{\mu}$ is an average cosine over distribution of light in sea depth, it is expressed through the inherent optical properties $a$ and $b_B$ or Gordon’s parameter $g = b_B/(a + b_B)$ as follows:

$$\bar{\mu} = \frac{a}{a + 3b_B + \sqrt{b_B(4a + 9b_B)}} \equiv \frac{1 - g}{\sqrt{1 + 2g + \sqrt{g}(4 + 5g)}},$$

(2)

The source functions $f_1$ and $f_2$ are expressed through the distribution of the light radiance just below the sea surface $L_q(\mu, \phi)$ as follows:

$$f_1(z) = b \int_{0}^{2\pi} \int_{0}^{1} 2B - \psi(\mu) L_q(\mu, \phi) \exp(-\alpha z / \mu) d\mu d\phi,$$

(3)

$$f_2(z) = b \int_{0}^{2\pi} \int_{0}^{1} \psi(\mu) L_q(\mu, \phi) \exp(-\alpha z / \mu) d\mu d\phi,$$

(4)

$$\psi(\mu) = \frac{1}{2} \int_{0}^{1} p(-\mu', \mu) d\mu', \quad p(\mu, \mu') = \frac{1}{2\pi} \int_{0}^{2\pi} p(\cos \gamma) d\phi,$$

(5)

$$\cos \gamma = \mu \mu' + \sqrt{1 - \mu^2} \sqrt{1 - \mu'^2} \cos(\phi - \phi'), \quad \mu = \cos \theta, \quad \mu' = \cos \theta',$$

(6)

Here $\alpha = a + 2b_B$ is a renormalized attenuation coefficient, $\theta$ is a zenith angle measured from the 0z axis, $\phi$ is an azimuthal angle measured from the direction to the Sun, $p(\cos \gamma)$ is a scattering phase function, $\gamma$ is a scattering angle, $B$ is a probability of backscattering:

$$B = \frac{1}{2} \int_{\pi/2}^{\pi} p(\cos \theta) \sin \theta d\theta \equiv \frac{1}{2} \int_{-1}^{0} p(\mu) d\mu.$$

(7)

The total downward $E_d$ and upward $E_u$ irradiances are determined by

$$E_d(z) = \int_{0}^{2\pi} \int_{0}^{1} \mu L_q(\mu, \phi) d\mu d\phi + E_1(z), \quad E_u(z) = E_2(z).$$

(8)

We can represent Eqs. (1) in the following matrix form:

$$\hat{L}_{ik}(z)E_k(z) = f_i(z), \quad (i = 1, 2),$$

(9)

where the repeated indices imply summation and the differential matrix operator $\hat{L}_{ik}$ is given by the following equation:
\[
\hat{L}_{ik}(z) = \begin{bmatrix} (2 - \bar{\mu})(a + b) + \frac{d}{dz} & -(2 + \bar{\mu})b \\ -(2 - \bar{\mu})b & (2 + \bar{\mu})(a + b) - \frac{d}{dz} \end{bmatrix}.
\] (10)

The solution to Eq. (9) is sought as a sum of general and partial solutions: 4

\[
E_i(z) = A_i \exp(-\alpha_{-\infty} z) + E_i e^{\exp(\alpha_0 z)} + \int_{-\infty}^{+\infty} G_{ik}(z - z') f_i(z') dz',
\] (11)

here \( -\alpha_{-\infty} \) and \( \alpha_0 \) are eigenvalues of the system of equations (9) or (1); \( \alpha_{-\infty} \) has physical meaning of a diffuse attenuation coefficient,

\[
\alpha_{-\infty} = \frac{a}{\bar{\mu}}, \quad \alpha_0 = b(2 - \bar{\mu})\left(\frac{1}{R_{\infty}} - R_0\right) - \frac{a}{\bar{\mu}},
\] (12)

\[
R_{\infty} = \left(\frac{1 - \bar{\mu}}{1 + \bar{\mu}}\right)^2, \quad R_0 = \frac{2 + \bar{\mu}}{2 - \bar{\mu}} R_{\infty} = \left(\frac{2 + \bar{\mu}}{2 - \bar{\mu}}\right)\left(\frac{1 - \bar{\mu}}{1 + \bar{\mu}}\right)^2,
\] (13)

here \( R_{\infty} \) is a diffuse reflection coefficient of deep sea illuminated by diffuse light, \( a_i = (1,R_{\infty}), \) \( e_i = (R_0,1), \) the constants \( A \) and \( E \) are determined by boundary conditions, and \( G_{ik} \) is Green’s matrix of Eq. (9). The Green matrix \( G_{ik} \) is a solution of the following equation:

\[
\hat{L}_{ik}(z) G_{kj}(z) = \delta_{ij} \delta(z),
\] (14)

here \( \delta_{ik} \) is the Kronecker symbol or unity matrix, and \( \delta(z) \) is the Dirac delta-function. 4

We can represent the Green matrix \( G_{ik} \) through a scalar Green’s function \( G \) using following relationships:

\[
G_{11}(z) = \left[(2 + \bar{\mu})(a + b) - \frac{d}{dz}\right] D(z), \quad G_{12}(z) = (2 + \bar{\mu})b D(z), \\
G_{21}(z) = (2 - \bar{\mu})b D(z), \quad G_{22}(z) = (2 - \bar{\mu})(a + b) + \frac{d}{dz} D(z).
\] (15)

The scalar Green function \( G \) satisfies the following equation:

\[
\left[-\frac{d^2}{dz^2} + 2\bar{\mu}(a + b) \frac{d}{dz} + a(4 - \bar{\mu}^2)(a + 2b)\right] G(z) = \delta(z).
\] (16)

The solution to this equation is easily obtained and has the following form:
\[
G(z) = \frac{H(z) \exp(-\alpha_\infty z) + H(-z) \exp(\alpha_0 z)}{\alpha_\infty + \alpha_0}, \quad H(z) = \begin{cases} 
1, & z > 0, \\
0, & z \leq 0,
\end{cases} \quad \frac{dH(z)}{dz} = \delta(z). \tag{17}
\]

The function \( H \) in Eq. (17) is the Heavyside or step function. By substituting Eq. (17) into Eqs. (15) we obtain the following expression for the Green matrix \( G_{ik} \):

\[
G_{ik}(z) = \left[ \frac{R_0}{R_i R_\infty} \int_{-\infty}^{\infty} H(z) \exp(-\alpha_\infty z) dz \right] + \left[ \frac{R_0}{R_i R_\infty} \int_{-\infty}^{\infty} H(-z) \exp(\alpha_0 z) dz \right]. \tag{18}
\]

Equations (11) and (18) allow to obtain solutions to Eq. (1) or (9) for the case of arbitrary source functions.

For the optically deep ocean our approximation gives:

\[
f_i(z) = f_2(z) = b_y F(z), \quad F(z) = \int_0^{2\pi} \int_0^1 L_y(\mu, \varphi) \exp(-\alpha z / \mu) d\mu.
\tag{19}
\]

By applying boundary condition at \( z = +0, \ E_i(0) = 0 \), we obtain the following solutions for the downward and upward irradiances of scattered light:

\[
E_1(z) = \frac{b_y (1 + R_\infty)}{1 - R_i R_\infty} \left[ \frac{R_0}{R_i R_\infty} \int_{-\infty}^{\infty} F(z') \exp(-\alpha_\infty (z - z')) dz' + R_0 \int_{-\infty}^{\infty} F(z') \exp(\alpha_0 z - \alpha_\infty z') dz' - \int_{-\infty}^{\infty} F(z') \exp(-\alpha_0 z') dz' \right], \tag{20}
\]

\[
E_2(z) = \frac{b_y (1 + R_\infty)}{1 - R_i R_\infty} \left[ \frac{R_0}{R_i R_\infty} \int_{-\infty}^{\infty} F(z') \exp(-\alpha_\infty (z - z')) dz' + \left[ \int_{-\infty}^{\infty} F(z') \exp(\alpha_0 (z - z')) dz' - R_0 R_\infty \int_{-\infty}^{\infty} F(z') \exp(-\alpha_0 z') dz' \right] \right]. \tag{21}
\]

The total downward and upward irradiances are obtained as a sums of diffuse irradiances given by Eqs. (20) and (21) and direct irradiances given by Eqs. (8):

\[
E_d(z) = \int_0^{2\pi} \mu d\mu \int_0^{2\pi} L_y(\mu, \varphi) d\phi + \frac{b_y}{1 - R_i R_\infty} \int_0^{2\pi} \mu d\mu \int_0^{2\pi} L_y(\mu, \varphi) d\phi \left[ \frac{1 + R_0}{\alpha - \alpha_\infty \mu} - R_0 (1 + R_\infty) \frac{e^{\alpha_0 z} - e^{-\alpha z / \mu}}{\alpha + \alpha_0 \mu} \right]. \tag{22}
\]
According to the definition $R = E_a(0)/E_u(0)$. By using Eqs. (22) and (23) we find that the diffuse reflection coefficient of the ocean is:

$$R = g\left(\frac{1 + R_\infty}{1 + g}\right) \int_0^1 \mu d\mu \int_0^{2\pi} L_q(\mu, \varphi) d\varphi \int_0^{2\pi} L_q(\mu, \varphi) d\varphi, \quad (24)$$

here $L_q(\mu, \varphi)$ is an angular distribution of light radiance just below the ocean surface,

$$\varepsilon = \eta + \sqrt{g^2\eta^2 + 4\left(\frac{1 - g}{1 + g}\right)}, \quad \eta = \sqrt{\frac{1 + 2g - \sqrt{g(4 + 5g)}}{(1 + g)^2}}, \quad R_\infty = \left(\frac{1 - \bar{\mu}}{1 + \bar{\mu}}\right)^2. \quad (25)$$

$$g = \frac{(1 - \bar{\mu}^2)^2}{1 + \bar{\mu}^2(4 - \bar{\mu}^2)} \equiv \frac{b_B}{a + b_B} \equiv \frac{\omega_0 B}{1 - \omega_0 + \omega_0 B}, \quad \omega_0 = \frac{b}{a + b}, \quad (26)$$

here $\omega_0$ is a single scattering albedo and $b$ is a scattering coefficient ($b_B = bB$). The radiance distribution $L_q(\mu, \varphi)$ is determined by the radiative transfer in atmosphere and is defined below.

In the simplest case of homogeneous radiance distribution $L_q(\mu, \varphi) = \text{const}$, we have the following formula for diffuse reflection coefficient

$$R = \frac{2g}{\varepsilon} \left(\frac{1 + R_\infty}{1 + g}\right) \left[1 - \frac{1}{\varepsilon} \ln(1 + \varepsilon)\right], \quad (27)$$

and for the case of directed illumination $L_q(\mu, \varphi) = L_q(0) \delta(\mu - \mu_0) \delta(\varphi)$, the diffuse reflection coefficient is

$$R = \frac{g}{1 + g} \frac{1 + R_\infty}{1 + \varepsilon \mu_0}. \quad (28)$$
3. EQUATIONS FOR IRRADIANCES IN THE ATMOSPHERE

Let us place the origin of cartesian coordinates on the height $H$ just below the absorbing ozone layer and direct the $0z$ axis towards the ocean surface. In this case we can consider the atmosphere below the ozone layer as unabsorbing and purely scattering. If the irradiance by the sunlight above the ozone layer is $S_0$, then the irradiance by the sunlight below ozone layer is:

$$E_0 = S_0 T_{oz}, \quad T_{oz} = \exp(-\tau_{oz} / \mu_s),$$

(29)

where $T_{oz}$ is a transmittance by the ozone layer, and $\tau_{oz}$ is an ozone absorption optical depth, $\mu_s = \cos z_s$, $z_s$ is the solar zenith angle.

Similarly to Eqs. (1) we can write the following system of equations for downward $E_1^a$ and upward $E_2^a$ scattered light irradiances in the atmosphere:

$$\left\{ \begin{array}{l}
\frac{d}{d\tau} + 2 B_a \left[ E_1^a(\tau) - 2 B_a E_2^a(\tau) \right] = f_1^a(\tau), \\
-2 B_a E_1^a(z) + \left( -\frac{d}{d\tau} + 2 B_a \right) E_2^a(z) = f_2^a(\tau),
\end{array} \right.$$

(30)

$$\tau = \int_z^H \sigma(z') dz'$$

is the optical thickness between heights $z$ and $H$, $\sigma(z)$ is the scattering coefficient of an atmospheric air at height $z$, $B_a$ is the probability of scattering of light in atmospheric air,

$$B_a = \frac{1}{2} \int_{-1}^1 p_a(\mu) d\mu, \quad p_a(\mu) = \frac{\tau_R p_R(\mu) + \tau_A p_A(\mu)}{\tau_R + \tau_A},$$

(31)

$p_a(\mu)$ is the atmospheric phase function of scattering, $p_R(\mu)$ and $p_A(\mu)$ are, respectively, Rayleigh and aerosol phase function of scattering, $\tau_R$ and $\tau_A$ are, respectively, Rayleigh and aerosol atmospheric optical thicknesses, $f_1^a$ and $f_2^a$ are source functions to be determined later.

Similar to §2 the forward scattered light is included into unscattered component; and the renormalized attenuation coefficient in respect to $\tau$ is equal to $2 B_a$ ($a$ of §2 with $a = 0$). We also include the sunlight reflected from the sea surface into the direct light (the light of the sources), in this case we have the following expression for the solar radiance in the atmosphere below the ozone layer:

$$S(\tau, \mu, \phi) = E_0 e^{-2 B_a \tau / \mu_s} \delta(\phi) \delta(\mu - \mu_s) + E_0 R_F(z_s) e^{-2 B_a (2 \tau^* - \tau) / \mu_s} \delta(\pi - \phi) \delta(\mu + \mu_s),$$

(32)

here $R_F(z_s)$ is a Fresnel reflection coefficient of sunlight from the ocean surface,

$$\tau^* = \int_0^H \sigma(z) dz = \tau_R + \tau_A,$$

(33)

is the total optical thickness of the atmosphere below the ozone layer, $\delta(x)$ is Dirac’s delta
function, $\mu = \cos \theta$, $\theta$ is a zenith angle that is measured from the $0z$ axis, $\phi$ is an azimuth angle measured from the Sun. The downward $E_1^s$ and upward $E_2^s$ unscattered radiances are determined by the following formulae:

$$E_1^s(\tau) = \int_0^{2\pi} d\phi \int_0^1 S(\tau, \mu, \phi) \mu d\mu = E_0 \mu_s e^{-2B_2/\mu_s}, \quad (34)$$

$$E_2^s(\tau) = -\int_0^{2\pi} d\phi \int_0^{-1} S(\tau, \mu, \phi) \mu d\mu = E_0 R_F(z_s) \mu_s e^{-2B_2(2\tau^*-\tau)/\mu_s}. \quad (35)$$

The source functions in Eq. (30) are determined by

$$f_i^a(\tau) = f_i^a(\tau) = B_a F_a(\tau), \quad (36)$$

$$F_a(\tau) = \int_0^{2\pi} d\phi \int_0^1 S(0, \mu, \phi) e^{-2B_a \tau/\mu} d\mu = E_0 \left[ e^{-2B_a \tau/\mu} + R_F(z_s) e^{-2B_2(2\tau^*-\tau)/\mu_s} \right]. \quad (37)$$

System of Eqs. (30) here reduces to the Eqs. (9) with the following differential operator

$$\hat{L}_{ik}(\tau) = \begin{pmatrix} 2B_a + \frac{d}{d\tau} & -2B_a \\ -2B_a & 2B_a - \frac{d}{d\tau} \end{pmatrix}, \quad (38)$$

The solution of Eqs. (30) [or Eqs. (9) with the operator $\hat{L}_{ik}$ given by Eq. (38)] is:

$$E_i(\tau) = P_i + C_i \tau + \int_{-\infty}^{+\infty} D_{ik}(\tau - \tau') f_k^a(\tau') d\tau', \quad i, k = 1, 2, \quad (39)$$

here $C_1 = C_2 = C$, $P_1 = P$, $P_2 = P + C / (2B_a)$ are integration constants.

Green's matrix $D_{ik}$ is determined by the following equation:

$$\hat{L}_{ik}(\tau) D_{ik}(\tau) = \delta_{ik} \delta(\tau) \quad (40)$$

Using the approach of §2 we have for the $D_{ik}$ matrix:

$$D_{ik}(\tau) = \frac{1}{2} \text{sign} \tau \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} B_a |\tau| \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (41)$$

Values of constants $P$ and $C$ are determined by the boundary conditions just below the ozone layer ($\tau = 0$) and on the ocean surface ($\tau = \tau^*$):
\[ E_1^0(0) = 0, \quad E_2^0(\tau^*) = \left( A_s + R_F^D \right) E_1^0(\tau^*) + A_s E_1^0(\tau^*) \]  

(42)

here \( A_s \) is an albedo of the sea, \( R_F^D \) is a Fresnel reflection coefficient of the diffuse light:

\[ R_F^D = 2 \int_0^1 R_F(\mu) \mu \, d\mu . \]  

(43)

By inserting Eq. (41) into Eqs. (39) and (42) we have the following expressions for the integration constants:

\[ P = \frac{B_a}{2} \int_0^{\tau^*} F(\tau') \, d\tau' + 2 B_a^2 \int_0^{\tau^*} F(\tau') \tau' \, d\tau' , \]  

(44)

\[ C = \frac{A_s E_1^0(\tau^*) + B_a \left[ A_s + R_F^D - 2 B_a \tau^* \left( 1 + A_s + R_F^D \right) \right] \int_0^{\tau^*} F_a(\tau') \, d\tau'}{\tau^* \left[ 1 - A_s - R_F^D + 1/(2 B_a \tau^*) \right]} . \]  

(45)

The downward and upward irradiances of scattered light are

\[ E_1^a(\tau) = C \tau + B_a \int_0^{\tau^*} F_a(\tau') \, d\tau' + 2 B_a^2 \int_0^{\tau^*} \left[ \tau' - \tau \right] F_a(\tau') \, d\tau', \]  

(46)

\[ E_2^a(\tau) = \frac{C}{2 B_a} + C \tau - B_a \int_0^{\tau^*} F_a(\tau') \, d\tau' + B_a \int_0^{\tau^*} \left[ 1 + 2 B_a (\tau' - \tau) \right] F_a(\tau') \, d\tau' . \]  

(47)

The total atmospheric radiances are:

\[ E_1^a(\tau) = E_1^0(\tau) + E_1^a(\tau), \]  

(48)

\[ E_2^a(\tau) = E_2^0(\tau) + E_2^a(\tau), \]  

(49)

and the albedo of the ocean-atmosphere system below the ozone layer is determined by:

\[ A_{oa}^0 = E_1^0(0) / E_2^0(0) \equiv E_2^0(0) / E_1^0(0) = R_F(z_o) e^{-4 B_o \tau^* / \mu_s} + E_2^0(0) / (E_0 \mu_s) . \]  

(50)

The albedo of the whole ocean-atmosphere system is:

\[ A_{oa} = R_F(z_o) T_{oc}^2 e^{-4 B_o \tau^* / \mu_s} + T_{oc}^D e^D(0) / (E_0 \mu_s) , \]  

(51)

here \( T_{oc}^D \) is the transmission of diffuse light by the ozone layer.
4. ALBEDO OF THE OCEAN-ATMOSPHERIC SYSTEM

The sea albedo $A_S$ is determined by the formula:

$$A_S = T_d T_u R \left[ 1 - (1 - T_u) R \right] = T_d^2 R \left[ n_w^2 - (n_w^2 - T_d) R \right],$$

where $T_d$ is a transmission coefficient of the downward irradiance from the atmosphere to the sea, $T_u$ is a transmission coefficient of the upward irradiance from the sea to the atmosphere, they are connected with each other by the formula: $n_w^2 T_u = T_d$, where $n_w$ is a water refraction index, and $R$ is a diffuse reflection coefficient of the sea given by Eq. (24). The downward transmission coefficient is determined by the following equation:

$$T_d = \frac{1 - R_F(z_s)}{E'_i(\tau^*)} \left[ 1 - R_F(z_s) \right] E_i(\tau^*) = \frac{1 - R_F(z_s) + \left[ 1 - R_F(z_s) \right] q}{1 + q},$$

where

$$q = \frac{E'_a(\tau^*)}{E'_i(\tau^*)} = R_a \frac{A_s}{1 - R_a(A_s + R_F^D)} +$$

$$+ R_a \left[ 1 - 4 B_a \tau + (1 - 2 B_a \tau) (A_s + R_F^D) \right] \left[ e^{2 B_a \tau^*} - 1 \right] \left[ 1 + R_F(z_s) e^{-2 B_a \tau^*} \right] \left[ 1 - R_a(A_s + R_F^D) \right].$$

To obtain $A_S$ we should solve a system of equations (54)-(56). Fortunately, in real life situation $q << 1$ and may be neglected. In this case the value for sea albedo may be estimated by formula

$$A_S \equiv \left[ 1 - R_F(z_s) \right]^2 R \left( n_w^2 - \left[ 1 - R_F(z_s) \right] R \right).$$

By inserting value $F(\tau)$ and $E'_i(\tau^*)$ from Eqs. (37) and (34) we have the following equation for the albedo of the ocean-atmosphere measured above the ozone layer:

$$A_{oa} = R_F(z_s) T_{oc}^2 e^{-4 B_a \tau^*/\mu_s} + \frac{T_{oc} T_d A_S}{1 - 2 R_a(A_s + R_F^D)} e^{-2 B_a \tau^*/\mu_s} +$$

$$+ T_{oc}^D \frac{T_a + (T_a - 2 R_a) (A_s + R_F^D)}{2 \left[ 1 - 2 R_a(A_s + R_F^D) \right]} \left( 1 - e^{-2 B_a \tau^*/\mu_s} \right) \left[ 1 + R_F(z_s) e^{-2 B_a \tau^*/\mu_s} \right],$$

$$T_{oc}^D = 2 \int_0^1 e^{-\tau_{oc}/\mu} \mu d\mu = 2 \tau_{oc}^2 \int_{\tau_{oc}}^{\infty} e^{-z^3} dz,$$

$$E_a^2(0) = \frac{A_s E'_i(\tau^*) + B_a \left[ 1 + (1 - 4 B_a \tau^*) (A_s + R_F^D) \right] \int_0^{\tau_F} F_a(\tau') d\tau'}{\left[ 1 + 2 B_a \tau^* - 2 B_a \tau^* (A_s + R_F^D) \right]}.$$
5. MODEL OF SEAWATER OPTICAL PROPERTIES

In order to calculate diffuse reflectance coefficient of the ocean we need to know seawater inherent optical properties. Here we adopt a one-parameter model that connects absorption and backscattering coefficients $a$ and $b_B$ of seawater with the concentration of chlorophyll $C_c$. This model is tuned to reproduce the same color indices that are used for remote restoration of chlorophyll content from satellite measurements.

The absorption coefficient of seawater $a(\lambda)$, ($m^{-1}$) is taken to be:

$$a(\lambda) = a_w(\lambda) + a_c(\lambda) + a_f(\lambda) + a_h(\lambda), \quad a_c(\lambda) = a_c^0(\lambda) \left( \frac{C_c}{C_c^0} \right)^{0.602}, \quad a_f(\lambda) = a_f^0 C_f \exp(-k_f \lambda), \quad a_h(\lambda) = a_h^0 C_h \exp(-k_h \lambda),$$

where $a_w(\lambda)$ is the pure water absorption coefficient in $m^{-1}$, $\lambda$ is the vacuum wavelength of light in $nm$, $a_c^0(\lambda)$ is the specific absorption coefficient of chlorophyll in $m^{-1}$, $C_c$ is the total concentration of chlorophyll in $mg/m^3$ ($C_c^0 = 1mg/m^3$), $a_f^0 = 35.959 m^2/mg$ is the specific absorption coefficient of fulvic acid (the first component of the colored dissolved organic matter (CDOM) or yellow substance); $k_f = 0.0189 nm^{-1}$; $a_h^0 = 18.828 m^2/mg$ is the specific absorption coefficient of humic acid (the second component of CDOM); $k_h = 0.01105 nm^{-1}$; $C_f$ and $C_h$ are, respectively, the concentrations of fulvic and humic acids in $mg/m^3$.

The backscattering ($b_B(\lambda)$) coefficient is calculated according to the equation:

$$b_B(\lambda) = 0.5 b_w(\lambda) + B_s b_s^0(\lambda) C_s + B_l b_l^0(\lambda) C_l,$$

where $B_s = 0.039$ and $B_l = 6.4 \cdot 10^{-4}$ are probabilities of backscattering on small particles ($B_s$), and large particles ($B_l$), $b_w(\lambda)$ is the scattering coefficient of pure water in $m^{-1}$, $b_s^0(\lambda)$ and $b_l^0(\lambda)$ are the specific scattering coefficients in $m^2/g$ for small and large particulate matter respectively, $C_s$ and $C_l$ are the concentrations in $g/m^3$ of small and large particles respectively. The equation for $b_w(\lambda)$ is derived by interpolating the data by Morel and Prieur:

$$b_w(\lambda) = (5.826 \cdot 10^{-3} m^{-1})(400/\lambda)^{1.322}.$$

The spectral dependencies for scattering coefficients of small and large particulate matter are given by the following equations:

$$b_s^0(\lambda) = (1.1513 m^2/g)(400/\lambda)^{1.7}, \quad b_l^0(\lambda) = (0.3411 m^2/g)(400/\lambda)^{0.3}$$
Concentrations $C_c$, $C_h$, $C_f$, $C_s$ and $C_l$ of dissolved and suspended matter are connected with the chlorophyll concentration $C_c$ as follows:

$$
C_f = 0.782 \cdot C_c \cdot \exp(0.800 + 0.123 \cdot C_c),\quad C_h = 0.337 \cdot C_c \cdot \exp(-0.554 + 0.123 \cdot C_c),
$$
$$
C_s = 0.153 \cdot C_c \cdot \exp(-2.177 + 0.116 \cdot C_c),\quad C_l = 0.575 \cdot C_c \cdot \exp(0.283 + 0.031 \cdot C_c).
$$

Equations (60)–(64) allow us to compute inherent optical properties of seawater as the functions of wavelength and chlorophyll concentration $C_c$.

6. MODEL OF ATMOSPHERIC OPTICAL PROPERTIES

The atmospheric optical thickness $\tau^*$ and probability of backscattering on atmospheric air $B_a$ are modeled according to Refs. 14, 15. The total atmospheric optical thickness is given by

$$
\tau^*(\lambda) = \tau_R(\lambda) + \tau_A(\lambda),\quad \tau_R(\lambda) = 0.36\left(\frac{400}{\lambda}\right)^{0.086},
$$

here $\tau_r(\lambda)$ is the optical thickness of the Rayleigh component of the atmosphere 15.

According to Ref. 15 the aerosol optical thickness $\tau_A$ may be expressed through the following version of the Ångstrom law:

$$
\tau_A(\lambda) = \tau_0 \left(\frac{745}{\lambda}\right)^{0.08} \tau_0 = \tau_a(745).
$$

The backscattering probability is defined by the following equation: 15

$$
B_a(\lambda) = \frac{(0.5 - \tau_0)\tau_A(\lambda) + 0.5\tau_R(\lambda)}{\tau_A(\lambda) + \tau_R(\lambda)}.
$$

Equations (65)-(67) express all atmospheric optical parameters of §4 through one parameter, aerosol atmospheric optical thickness at near infrared $\tau_0 \equiv \tau_a(745)$.

7. CONCLUSIONS

The unified approach to solve the light scattering problem in ocean-atmospheric system is presented. The analytic expression for the albedo of the ocean-atmospheric system is calculated. The one-parameter optical models of seawater and atmosphere are presented. The proposed model allows to calculate spectral albedo of the ocean-atmosphere system as a function of three input parameters: the chlorophyll concentration, the aerosol atmospheric optical thickness at near infrared, and the solar zenith angle.

8. ACKNOWLEDGMENTS

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9. REFERENCES


5. Fragments of sea water optical model may be found in Refs.3, 17. The final version is presented in Ref. 6.


14. Some fragments of atmospheric optical model used here may be found in Refs.7, 18. The final version is presented in Ref.15.


