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Matrix Equation for Radiative Transport in Sea-Atmospheric System

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ABSTRACT

The proposed approach is based on the theory developed for the marine environment. This approach is tuned to be valid in the whole range of optical properties including an unrealistic case of a totally scattering sea. Because the atmosphere below the ozone layer is totally scattering in the visible range of spectrum, the proposed approach may be applied to the atmosphere by setting the absorption coefficient to zero. The solutions to the light scattering problem for oceanic and atmospheric parts are obtained. The one-parameter optical models of seawater and atmosphere are presented. The proposed model allows to calculate spectral albedo of the ocean-atmosphere system as a function of three input parameters: the chlorophyll concentration, the aerosol atmospheric optical thickness at near infrared, and the solar zenith angle.

Keywords: radiative transfer, scattering, ocean optics, atmospheric optics

1. INTRODUCTION

Optical properties of the sea and the clear atmosphere are very different. Unclouded atmosphere without fog is always optically thin in the vertical direction. The sea, with the exception of some rare cases of clean and very shallow waters, is always optically thick. This difference usually results in implementing different approaches to calculate light fields in these environments. Real-time atmospheric correction algorithms may be improved with respect to the speed of calculation, if the underlying theories for atmospheric and oceanic light propagation are unified. This work presents a unified approach to light propagation in ocean-atmospheric environment that may be useful for some remote sensing applications.

2. EQUATIONS FOR IRRADIANCES IN THE OCEAN

Let us consider a homogeneous scattering and absorbing ocean illuminated by the light of Sun and sky. The system of equations for downward E_1 and upward E_2 irradiances of scattered light is written as follows:[1-3]

$$\left.\begin{aligned} \left[\frac{d}{dz}+(2-\bar{\mu})(a+b_B)\right]E_1(z)-(2+\bar{\mu})\,b_B\,E_2(z)&=f_1(z),\\ -(2-\bar{\mu})\,b_B\,E_1(z)+\left[-\frac{d}{dz}+(2+\bar{\mu})(a+b_B)\right]E_2(z)&=f_2(z).\end{aligned}\right\}\qquad(1)$$

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The cartesian coordinate system here is chosen in such a way that the *0z* axis is directed from the sea surface to the bottom. Here a is an absorption coefficient, b_B is a backscattering coefficient, $\overline{\mu}$ is an average cosine over distribution of light in sea depth, it is expressed through the inherent optical properties a and b_B or Gordon's parameter $g = b_B/(a+b_B)$ as follows:

$$\overline{\mu} = \sqrt{\frac{a}{a+3b_B+\sqrt{b_B(4a+9b_B)}}} \equiv \sqrt{\frac{1-g}{1+2g+\sqrt{g(4+5g)}}}, \tag{2}$$

The source functions f_1 and f_2 are expressed through the distribution of the light radiance just below the sea surface $L_q(\mu,\varphi)$ as follows:

$$f_1(z) = b\int_0^{2\pi} d\varphi \int_0^1 \left[2B-\psi(\mu)\right] L_q(\mu,\varphi)\exp(-\alpha z/\mu)\,d\mu\,, \tag{3}$$

$$f_2(z) = b\int_0^{2\pi} d\varphi \int_0^1 \psi(\mu)\, L_q(\mu,\varphi)\exp(-\alpha z/\mu)\,d\mu\,, \tag{4}$$

$$\psi(\mu) = \frac{1}{2}\int_0^1 p(-\mu',\mu)\,d\mu'\,, \quad p(\mu,\mu') = \frac{1}{2\pi}\int_0^{2\pi} p(\cos\gamma)\,d\varphi \tag{5}$$

$$\cos\gamma = \mu\,\mu' + \sqrt{1-\mu^2}\,\sqrt{1-\mu'^2}\,\cos(\varphi-\varphi'), \quad \mu = \cos\theta, \quad \mu' = \cos\theta', \tag{6}$$

here $\alpha = a + 2b_B$ is a renormalized attenuation coefficient, θ is a zenith angle measured from the *0z* axis, φ is an azimuthal angle measured from the direction to the Sun, $p(\cos\gamma)$ is a scattering phase function, γ is a scattering angle, B is a probability of backscattering:

$$B = \frac{1}{2}\int_{\pi/2}^{\pi} p(\cos\theta)\sin\theta\,d\theta \equiv \frac{1}{2}\int_{-1}^{0} p(\mu)\,d\mu\,. \tag{7}$$

The total downward E_d and upward E_u irradiances are determined by

$$E_d(z) = \int_0^{2\pi} d\varphi \int_0^1 \mu\, L_q(\mu,\varphi)\,d\mu + E_1(z), \quad E_u(z) = E_2(z). \tag{8}$$

We can represent Eqs. (1) in the following matrix form:

$$\hat{L}_{ik}(z)E_k(z) = f_i(z), \quad (i = 1,2), \tag{9}$$

where the repeated indices imply summation and the differential matrix operator $\hat{L}_{ik}$ is given by the following equation:

$$\hat{L}_{ik}(z) = \begin{Vmatrix} (2-\bar{\mu})(a+b_B) + \dfrac{d}{dz} & -(2+\bar{\mu})b_B \\ -(2-\bar{\mu})b_B & (2+\bar{\mu})(a+b_B) - \dfrac{d}{dz} \end{Vmatrix}. \tag{10}$$

The solution to Eq. (9) is sought as a sum of general and partial solutions: [4]

$$E_i(z) = A\,a_i \exp(-\alpha_\infty z) + E\,e_i \exp(\alpha_0\, z) + \int_{-\infty}^{+\infty} G_{ik}(z-z')\, f_k(z')\, dz', \tag{11}$$

here $-\alpha_\infty$ and α_0 are eigenvalues of the system of equations (9) or (1); α_∞ has physical meaning of a diffuse attenuation coefficient,

$$\alpha_\infty = \frac{a}{\bar{\mu}}, \quad \alpha_0 = b_B(2-\bar{\mu})\left(\frac{1}{R_\infty} - R_0\right) - \frac{a}{\bar{\mu}}, \tag{12}$$

$$R_\infty = \left(\frac{1-\bar{\mu}}{1+\bar{\mu}}\right)^2, \quad R_0 = \frac{2+\bar{\mu}}{2-\bar{\mu}} R_\infty \equiv \left(\frac{2+\bar{\mu}}{2-\bar{\mu}}\right)\left(\frac{1-\bar{\mu}}{1+\bar{\mu}}\right)^2, \tag{13}$$

here R_∞ is a diffuse reflection coefficient of deep sea illuminated by diffuse light, $a_i = (1, R_\infty)$, $e_i = (R_0, 1)$, the constants A and E are determined by boundary conditions, and G_{ik} is Green's matrix of Eq. (9). The Green matrix G_{ik} is a solution of the following equation:

$$\hat{L}_{ik}(z) G_{kl}(z) = \delta_{il}\, \delta(z), \tag{14}$$

here δ_{ik} is the Kronecker symbol or unity matrix, and $\delta(z)$ is the Dirac delta-function. [4]
We can represent the Green matrix G_{ik} through a scalar Green's function G using following relationships:

$$\begin{gathered} G_{11}(z) = \left[(2+\bar{\mu})(a+b_B) - \frac{d}{dz}\right] D(z), \qquad G_{12}(z) = (2+\bar{\mu})\, b_B\, D(z), \\ G_{21}(z) = (2-\bar{\mu})\, b_B\, D(z), \qquad G_{22}(z) = (2-\bar{\mu})(a+b_B) + \frac{d}{dz} D(z). \end{gathered} \tag{15}$$

The scalar Green function G satisfies the following equation:

$$\left[-\frac{d^2}{dz^2} + 2\bar{\mu}(a+b_B)\frac{d}{dz} + a\left(4-\bar{\mu}^2\right)(a+2b_B)\right] G(z) = \delta(z). \tag{16}$$

The solution to this equation is easily obtained and has the following form:

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$$G(z)=\frac{H(z)\exp(-\alpha_\infty z)+H(-z)\exp(\alpha_0 z)}{\alpha_\infty+\alpha_0},\quad H(z)=\begin{cases}1, & z>0,\\ 0, & z\le 0,\end{cases}\quad \frac{d\,H(z)}{dz}=\delta(z). \qquad (17)$$

The function H in Eq. (17) is the Heavyside or step function.

By substituting Eq. (17) into Eqs. (15) we obtain the following expression for the Green matrix G_{ik}:

$$G_{ik}(z)=\begin{Vmatrix}1 & R_0\\ R_\infty & R_0R_\infty\end{Vmatrix}\frac{H(z)\exp(-\alpha_\infty z)}{1-R_0R_\infty}+\begin{Vmatrix}R_0R_\infty & R_0\\ R_\infty & 1\end{Vmatrix}\frac{H(-z)\exp(\alpha_0 z)}{1-R_0R_\infty}. \qquad (18)$$

Equations (11) and (18) allow to obtain solutions to Eq. (1) or (9) for the case of arbitrary source functions.

For the optically deep ocean our approximation gives:

$$f_1(z)=f_2(z)=b_BF(z),\quad F(z)=\int_0^{2\pi}d\varphi\int_0^1 L_q(\mu,\varphi)\exp(-\alpha z/\mu)\,d\mu. \qquad (19)$$

By applying boundary condition at $z=+0$, $E_1(0)=0$, we obtain the following solutions for the downward and upward irradiances of scattered light:

$$\begin{aligned}E_1(z)=\frac{b_B(1+R_\infty)}{1-R_0R_\infty}\Bigg\{&\frac{1+R_0}{1+R_\infty}\int_0^z F(z')\exp[-\alpha_\infty(z-z')]dz'+\\ &R_0\left[\int_z^\infty F(z')\exp[\alpha_0(z-z')]dz'-\int_0^\infty F(z')\exp(-\alpha_0 z')dz'\right]\Bigg\},\end{aligned} \qquad (20)$$

$$\begin{aligned}E_2(z)=\frac{b_B(1+R_\infty)}{1-R_0R_\infty}\Bigg\{&\frac{R_\infty(1+R_0)}{1+R_\infty}\int_0^z F(z')\exp[-\alpha_\infty(z-z')]dz'+\\ &\left[\int_z^\infty F(z')\exp[\alpha_0(z-z')]dz'-R_0R_\infty\int_0^\infty F(z')\exp(-\alpha_0 z')dz'\right]\Bigg\}.\end{aligned} \qquad (21)$$

The total downward and upward irradiances are obtained as a sums of diffuse irradiances given by Eqs. (20) and (21) and direct irradiances given by Eqs. (8):

$$\begin{aligned}E_d(z)=&\int_0^1\mu\,d\mu\int_0^{2\pi}L_q(\mu,\varphi)\,d\varphi+\\ &+\frac{b_B}{1-R_0R_\infty}\int_0^1\mu\,d\mu\int_0^{2\pi}L_q(\mu,\varphi)\,d\varphi\left[(1+R_0)\frac{e^{-\alpha_\infty z}-e^{-\alpha z/\mu}}{\alpha-\alpha_\infty\mu}-R_0(1+R_\infty)\frac{e^{\alpha_0 z}-e^{-\alpha z/\mu}}{\alpha+\alpha_0\,\mu}\right],\end{aligned} \qquad (22)$$

$$E_u(z) = b_B(1+R_\infty)\int_0^1 \frac{\mu\, d\mu \int_0^{2\pi} L_q(\mu,\varphi)\, d\varphi}{\alpha+\alpha_0\mu} + \tag{23}$$
$$+\frac{b_B}{1-R_0R_\infty}\int_0^1 \mu\, d\mu \int_0^{2\pi} L_q(\mu,\varphi)\, d\varphi\left[R_\infty(1+R_0)\frac{e^{-\alpha_\infty z}-e^{-\alpha z/\mu}}{\alpha-\alpha_\infty\mu} - (1+R_\infty)\frac{1-e^{-\alpha z/\mu}}{\alpha+\alpha_0\mu}\right].$$

According to the definition $R = E_u(0)/E_d(0)$. By using Eqs. (22) and (23) we find that the diffuse reflection coefficient of the ocean is:

$$R = g\left(\frac{1+R_\infty}{1+g}\right)\int_0^1 \frac{\mu\, d\mu}{1+\varepsilon\,\mu}\int_0^{2\pi} L_q(\mu,\varphi)\, d\varphi \Bigg/ \int_0^1 \mu\, d\mu \int_0^{2\pi} L_q(\mu,\varphi)\, d\varphi, \tag{24}$$

here $L_q(\mu,\varphi)$ is an angular distribution of light radiance just below the ocean surface,

$$\varepsilon = \eta + \sqrt{g^2\eta^2 + 4\left(\frac{1-g}{1+g}\right)}, \quad \eta = \sqrt{\frac{1+2g-\sqrt{g(4+5g)}}{(1+g)^3}}, \quad R_\infty = \left(\frac{1-\bar{\mu}}{1+\bar{\mu}}\right)^2. \tag{25}$$

$$g = \frac{(1-\bar{\mu}^2)^2}{1+\bar{\mu}^2(4-\bar{\mu}^2)} \equiv \frac{b_B}{a+b_B} \equiv \frac{\omega_0 B}{1-\omega_0+\omega_0 B}, \quad \omega_0 = \frac{b}{a+b}, \tag{26}$$

here ω_0 is a single scattering albedo and b is a scattering coefficient $\left(b_B = b\,B\right)$. The radiance distribution $L_q(\mu,\varphi)$ is determined by the radiative transfer in atmosphere and is defined below.

In the simplest case of homogeneous radiance distribution $L_q(\mu,\varphi) = const$, we have the following formula for diffuse reflection coefficient

$$R = \frac{2g}{\varepsilon}\left(\frac{1+R_\infty}{1+g}\right)\left[1-\frac{1}{\varepsilon}\ln(1+\varepsilon)\right], \tag{27}$$

and for the case of directed illumination $L_q(\mu,\varphi) = L_q^0\,\delta(\mu-\mu_0)\,\delta(\varphi)$, the diffuse reflection coefficient is

$$R = \left(\frac{g}{1+g}\right)\frac{1+R_\infty}{1+\varepsilon\,\mu_0}. \tag{28}$$

3. EQUATIONS FOR IRRADIANCES IN THE ATMOSPHERE

Let us place the origin of cartesian coordinates on the height H just below the absorbing ozone layer and direct the $0z$ axis towards the ocean surface. In this case we can consider the atmosphere below the ozone layer as unabsorbing and purely scattering. If the irradiance by the sunlight above the ozone layer is S_0, then the irradiance by the sunlight below ozone layer is:

$$E_0 = S_0\, T_{oz}, \quad T_{oz} = \exp(-\tau_{oz} / \mu_s), \tag{29}$$

here T_{oz} is a transmittance by the ozone layer, and τ_{oz} is an ozone absorption optical depth, $\mu_s = \cos z_s$, z_s is the solar zenith angle.

Similarly to Eqs. (1) we can write the following system of equations for downward E_1^a and upward E_2^a scattered light irradiances in the atmosphere:

$$\left.\begin{aligned} \left(\frac{d}{d\tau} + 2\,B_a\right) E_1^a(\tau) - 2\,B_a\, E_2^a(\tau) = f_1^a(\tau), \\ -2\,B_a\, E_1^a(z) + \left(-\frac{d}{d\tau} + 2\,B_a\right) E_2^a(z) = f_2^a(\tau), \end{aligned}\right\} \tag{30}$$

$\tau = \int_z^H \sigma(z')\,dz'$ is the optical thickness between heights z and H, $\sigma(z)$ is the scattering coefficient of an atmospheric air at height z, B_a is the probability of scattering of light in atmospheric air,

$$B_a = \frac{1}{2}\int_{-1}^{0} p_a(\mu)\,d\mu, \quad p_a(\mu) = \frac{\tau_R p_R(\mu) + \tau_A p_A(\mu)}{\tau_R + \tau_A}, \tag{31}$$

$p_a(\mu)$ is the atmospheric phase function of scattering, $p_R(\mu)$ and $p_A(\mu)$ are, respectively, Rayleigh and aerosol phase function of scattering, τ_R and τ_A are, respectively, Rayleigh and aerosol atmospheric optical thicknesses, f_1^a and f_2^a are source functions to be determined later.

Similar to §2 the forward scattered light is included into unscattered component; and the renormalized attenuation coefficient in respect to τ is equal to $2\,B_a$ (α of §2 with $a = 0$). We also include the sunlight reflected from the sea surface into the direct light (the light of the sources). in this case we have the following expression for the solar radiance in the atmosphere below the ozone layer:

$$S(\tau,\mu,\varphi) = E_0\, e^{-2B_a\tau/\mu_s}\delta(\varphi)\delta(\mu-\mu_s) + E_0\, R_F(z_s)\,e^{-2\,B_a\,(2\tau^*-\tau)/\mu_s}\delta(\pi-\varphi)\delta(\mu+\mu_s), \tag{32}$$

here $R_F(z_s)$ is a Fresnel reflection coefficient of sunlight from the ocean surface,

$$\tau^* = \int_0^H \sigma(z)\,dz \equiv \tau_R + \tau_A, \tag{33}$$

is the total optical thickness of the atmosphere below the ozone layer, $\delta(x)$ is Dirac's delta function, $\mu = \cos\theta$, θ is a zenith angle that is measured from the $0z$ axis, φ is an azimuth angle measured from the Sun. The downward E_1^s and upward E_2^s unscattered radiances are determined by the following formulae:

$$E_1^s(\tau) = \int_0^{2\pi} d\varphi \int_0^1 S(\tau,\mu,\varphi)\,\mu\,d\mu = E_0\,\mu_s\,e^{-2B\tau/\mu_s}, \tag{34}$$

$$E_2^s(\tau) = -\int_0^{2\pi} d\varphi \int_{-1}^0 S(\tau,\mu,\varphi)\,\mu\,d\mu = E_0\,R_F(z_s)\,\mu_s\,e^{-2B(2\tau^*-\tau)/\mu_s}. \tag{35}$$

The source functions in Eq. (30) are determined by

$$f_1^a(\tau) = f_2^a(\tau) = B_a\,F_a(\tau), \tag{36}$$

$$F_a(\tau) = \int_0^{2\pi} d\varphi \int_0^1 S(0,\mu,\varphi)\,e^{-2B_a\tau/\mu}\,d\mu = E_0\left[e^{-2B_a\tau/\mu_s} + R_F(z_s)\,e^{-2B_a(2\tau^*-\tau)/\mu_s}\right]. \tag{37}$$

System of Eqs. (30) here reduces to the Eqs. (9) with the following differential operator

$$\hat{L}_{ik}(\tau) = \begin{Vmatrix} 2B_a + \dfrac{d}{d\tau} & -2B_a \\ -2B_a & 2B_a - \dfrac{d}{d\tau} \end{Vmatrix}. \tag{38}$$

The solution of Eqs. (30) [or Eqs. (9) with the operator $\hat{L}_{ik}$ given by Eq. (38)] is:

$$E_i(\tau) = P_i + C_i\tau + \int_{-\infty}^{+\infty} D_{ik}(\tau-\tau')\,f_k^a(\tau')\,d\tau', \quad i,k = 1,2, \tag{39}$$

here $C_1 = C_2 = C$, $P_1 = P$, $P_2 = P + C/(2B_a)$ are integration constants.

Green's matrix D_{ik} is determined by the following equation:

$$\hat{L}_{ik}(\tau)\,D_{kl}(\tau) = \delta_{il}\,\delta(\tau) \tag{40}$$

Using the approach of §2 we have for the D_{ik} matrix:

$$D_{ik}(\tau) = \frac{1}{2}\,\mathrm{sign}\,\tau \begin{Vmatrix} 1 & 0 \\ 0 & -1 \end{Vmatrix} - B_a|\tau| \begin{Vmatrix} 1 & 1 \\ 1 & 1 \end{Vmatrix} \tag{41}$$

Values of constants P and C are determined by the boundary conditions just below the ozone layer $(\tau = 0)$ and on the ocean surface $(\tau = \tau^*)$:

$$E_1^a(0) = 0, \quad E_2^a(\tau^*) = \left(A_S + R_F^D\right) E_1^a(\tau^*) + A_S\, E_1^s(\tau^*) \tag{42}$$

here A_S is an albedo of the sea, R_F^D is a Fresnel reflection coefficient of the diffuse light:

$$R_F^D = 2\int_0^1 R_F(\mu)\,\mu\, d\mu\ . \tag{43}$$

By inserting Eq. (41) into Eqs. (39) and (42) we have the following expressions for the integration constants:

$$P = \frac{B_a}{2}\int_0^{\tau^*} F(\tau')\,d\tau' + 2\,B_a^2\int_0^{\tau^*} F(\tau')\,\tau'\,d\tau'\ , \tag{44}$$

$$C = \frac{A_S\, E_1^s(\tau^*) + B_a\left[A_S + R_F^D - 2\,B_a\,\tau^*\left(1 + A_S + R_F^D\right)\right]\int_0^{\tau^*} F_a(\tau')\,d\tau'}{\tau^*\left[1 - A_S - R_F^D + 1\big/\left(2\,B_a\,\tau^*\right)\right]}. \tag{45}$$

The downward and upward irradiances of scattered light are

$$E_1^a(\tau) = C\,\tau + B_a\int_0^{\tau} F_a(\tau')\,d\tau' + 2\,B_a^2\int_0^{\tau^*}\left[\tau' - |\tau - \tau'|\right] F_a(\tau')\,d\tau', \tag{46}$$

$$E_2^a(\tau) = \frac{C}{2\,B_a} + C\,\tau - B_a\int_0^{\tau} F_a(\tau')\,d\tau' + B_a\int_0^{\tau^*}\left[1 + 2\,B_a\left(\tau' - |\tau - \tau'|\right)\right] F_a(\tau')\,d\tau'. \tag{47}$$

The total atmospheric radiances are:

$$E_d^a(\tau) = E_1^s(\tau) + E_1^a(\tau), \tag{48}$$

$$E_u^a(\tau) = E_2^s(\tau) + E_2^a(\tau), \tag{49}$$

and the albedo of the ocean-atmosphere system below the ozone layer is determined by:

$$A_{oa}^0 = E_u^a(0)\,/\,E_d^a(0) \equiv E_u^a(0)\,/\,E_1^s(0) = R_F(z_s)\,e^{-4\,B_a\tau^*/\mu_s} + E_2^a(0)\big/\left(E_0\,\mu_s\right). \tag{50}$$

The albedo of the whole ocean-atmosphere system is:

$$A_{oa} = R_F(z_s)\,T_{oz}^2\,e^{-4\,B_a\tau^*/\mu_s} + T_{oz}T_{oz}^D\,E_2^a(0)\big/\left(E_0\,\mu_s\right), \tag{51}$$

here T_{oz}^D is the transmission of diffuse light by the ozone layer:

$$T_{oz}^{D} = 2\int_{0}^{1} e^{-\tau_{oz}/\mu}\,\mu\, d\mu = 2\,\tau_{oz}^{2}\int_{\tau_{oz}}^{\infty} e^{-z} z^{-3} dz, \tag{52}$$

$$E_2^a(0) = \frac{A_S\, E_1^s(\tau^*) + B_a\left[1+\left(1-4\,B_a\tau^*\right)\left(A_S + R_F^D\right)\right]\int_0^{\tau^*} F_a(\tau')\,d\tau'}{\left[1+2\,B_a\tau^* - 2\,B_a\tau^*(A_S + R_F^D)\right]} \tag{53}$$

4. ALBEDO OF THE OCEAN-ATMOSPHERIC SYSTEM

The sea albedo A_S is determined by the formula: [2]

$$A_S = T_d T_u\, R\Big/\left[1-\left(1-T_u\right)R\right] = T_d^2\, R\Big/\left[n_w^2 - \left(n_w^2 - T_d\right)R\right], \tag{54}$$

here T_d is a transmission coefficient of the downward irradiance from the atmosphere to the sea, T_u is a transmission coefficient of the upward irradiance from the sea to the atmosphere, they are connected with each other by the formula: $n_w^2\, T_u = T_d$, where n_w is a water refraction index, and R is a diffuse reflection coefficient of the sea given by Eq. (24). The downward transmission coefficient is determined by the following equation:

$$T_d = \frac{\left[1-R_F(z_s)\right]E_1^s(\tau^*) + \left[1-R_F^D\right]E_1(\tau^*)}{E_1^s(\tau^*) + E_1(\tau^*)} = \frac{\left[1-R_F(z_s)\right] + \left[1-R_F^D\right]q}{1+q}, \tag{55}$$

where

$$q = \frac{E_1^a(\tau^*)}{E_1^s(\tau^*)} = R_a\,\frac{A_S}{\left[1-R_a\left(A_S + R_F^D\right)\right]} + \\ + R_a\,\frac{\left[1-4\,B_a\tau^* + \left(1-2\,B_a\,\tau^*\right)\left(A_S + R_F^D\right)\right]\left(e^{2\,B_a\tau^*/\mu_s} - 1\right)\left[1+R_F(z_s)\,e^{-2\,B_a\tau^*/\mu_s}\right]}{2\left[1-R_a\left(A_S + R_F^D\right)\right]}. \tag{56}$$

To obtain A_S we should solve a system of equations (54)-(56). Fortunately, in real life situation $q \ll 1$ and may be neglected. In this case the value for sea albedo may be estimated by formula

$$A_S \cong \left[1-R_F(z_s)\right]^2 R\Big/\left(n_w^2 - \left\{n_w^2 - \left[1-R_F(z_s)\right]\right\}R\right). \tag{57}$$

By inserting value $F(\tau)$ and $E_1^s(\tau^*)$ from Eqs. (37) and (34)we have the following equation for the albedo of the ocean-atmosphere measured above the ozone layer:

$$A_{oa} = R_F(z_s)\, T_{oz}^2 e^{-4\,B_a\tau^*/\mu_s} + \frac{T_{oz}^D\, T_{oz}\, A_S\, T_a}{1-2\,R_a(A_S + R_F^D)}\, e^{-2\,B_a\tau^*/\mu_s} + \\ + T_{oz}^D\, T_{oz}\,\frac{T_a + \left(T_a - 2\,R_a\right)\left(A_S + R_F^D\right)}{2\left[1-2\,R_a(A_S + R_F^D)\right]}\left(1-e^{-2\,B_a\tau^*/\mu_s}\right)\left[1+R_F(z_s)\,e^{-2\,B_a\tau^*/\mu_s}\right], \tag{58}$$

here

$$T_a = \frac{1}{1+2B_a\tau^*}, \quad R_a = 1 - T_a = \frac{2B_a\tau^*}{1+2B_a\tau^*}. \tag{59}$$

5. MODEL OF SEAWATER OPTICAL PROPERTIES

In order to calculate diffuse reflectance coefficient of the ocean we need to know seawater inherent optical properties. [5] Here we adopt a one-parameter model [6, 7] that connects absorption and backscattering coefficients a and b_B of seawater with the concentration of chlorophyll C_c. This model is tuned to reproduce the same color indices that are used for remote restoration of chlorophyll content from satellite measurements.

The absorption coefficient of seawater $a(\lambda)$, (m^{-1}) is taken to be:

$$\left.\begin{array}{ll} a(\lambda) = \quad a_w(\lambda) + a_c(\lambda) + a_f(\lambda) + a_h(\lambda), & a_c(\lambda) = a_c^0(\lambda)\left(C_c/C_c^0\right)^{0.602}, \\ a_f(\lambda) = a_f^0\, C_f \exp(-k_f\,\lambda), & a_h(\lambda) = a_h^0\, C_h \exp(-k_h\,\lambda), \end{array}\right\}, \tag{60}$$

where $a_w(\lambda)$ is the pure water absorption coefficient in m^{-1} [8], λ is the vacuum wavelength of light in nm, $a_0^c(\lambda)$ is the specific absorption coefficient of chlorophyll in m^{-1} [9], C_c is the total concentration of chlorophyll in mg/m^3 $(C_c^0 = 1\,mg/m^3)$, $a_f^0 = 35.959\,m^2/mg$ is the specific absorption coefficient of fulvic acid (the first component of the colored dissolved organic matter (CDOM) or yellow substance); $k_f = 0.0189\,nm^{-1}$; $a_h^0 = 18.828\,m^2/mg$ is the specific absorption coefficient of humic acid (the second component of CDOM); $k_h = 0.01105\,nm^{-1}$; C_f and C_h are, respectively, the concentrations of fulvic and humic acids in mg/m^3.[10, 11]

The backscattering ($b_B(\lambda)$) coefficient is calculated according to the equation: [12]

$$b_B(\lambda) \;=\; 0.5\, b_w(\lambda) + B_s\, b_s^0(\lambda)\, C_s + B_l\, b_l^0(\lambda)\, C_l\ , \tag{61}$$

where $B_s = 0.039$ and $B_l = 6.4\cdot 10^{-4}$ are probabilities of backscattering on small particles (B_s), and large particles (B_l), $b_w(\lambda)$ is the scattering coefficient of pure water in m^{-1}, $b_s^0(\lambda)$ and $b_l^0(\lambda)$ are the specific scattering coefficients in m^2/g for small and large particulate matter respectively, C_s and C_l are the concentrations in g/m^3 of small and large particles respectively. The equation for $b_w(\lambda)$ is derived by interpolating the data by Morel and Prieur: [13]

$$b_w(\lambda) \;=\; \left(5.826\cdot 10^{-3}\, m^{-1}\right)\left(400/\lambda\right)^{4.322}. \tag{62}$$

The spectral dependencies for scattering coefficients of small and large particulate matter are given by the following equations: [12]

$$b_s^0(\lambda) \;=\; \left(1.1513\, m^2/g\right)\left(400/\lambda\right)^{1.7}, \quad b_l^0(\lambda) \;=\; \left(0.3411\, m^2/g\right)\left(400/\lambda\right)^{0.3} \tag{63}$$

Concentrations C_c, C_h, C_f, C_s and C_l of dissolved and suspended matter are connected with the chlorophyll concentration C_c as follows:

$$\left.\begin{aligned} C_f &= 0.782\cdot C_c\cdot\exp(\ 0.800+0.123\cdot C_c), \quad C_h = 0.337\cdot C_c\cdot\exp(-0.554+0.123\cdot C_c),\\ C_s &= 0.153\cdot C_c\cdot\exp(-2.177+0.116\cdot C_c), \quad C_l = 0.575\cdot C_c\cdot\exp(\ 0.283+0.031\cdot C_c).\end{aligned}\right\} \tag{64}$$

Equations (60)–(64) allow us to compute inherent optical properties of seawater as the functions of wavelength and chlorophyll concentration C_c.

6. MODEL OF ATMOSPHERIC OPTICAL PROPERTIES

The atmospheric optical thickness τ^* and probability of backscattering on atmospheric air B_a are modeled according to Refs. [14, 15]. The total atmospheric optical thickness is given by

$$\tau^*(\lambda) = \tau_R(\lambda) + \tau_A(\lambda), \quad \tau_R(\lambda) = 0.36(400/\lambda)^{4.086}, \tag{65}$$

here $\tau_R(\lambda)$ is the optical thickness of the Rayleigh component of the atmosphere [15].

According to Ref. [15] the aerosol optical thickness τ_A may be expressed through the following version of the Ångstrom law:

$$\tau_A(\lambda) = \tau_0 (745/\lambda)^{\frac{0.08}{\tau_0}} \qquad \tau_0 \equiv \tau_a(745). \tag{66}$$

The backscattering probability is defined by the following equation: [15]

$$B_a(\lambda) = \frac{(0.5-\tau_0)\tau_A(\lambda) + 0.5\tau_R(\lambda)}{\tau_A(\lambda)+\tau_R(\lambda)}. \tag{67}$$

Equations (65)-(67) express all atmospheric optical parameters of §4 through one parameter, aerosol atmospheric optical thickness at near infrared $\tau_0 \equiv \tau_a(745)$.

7. CONCLUSIONS

The unified approach to solve the light scattering problem in ocean-atmospheric system is presented. The analytic expression for the albedo of the ocean-atmospheric system is calculated. The one-parameter optical models of seawater and atmosphere are presented. The proposed model allows to calculate spectral albedo of the ocean-atmosphere system as a function of three input parameters: the chlorophyll concentration, the aerosol atmospheric optical thickness at near infrared, and the solar zenith angle.

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9. REFERENCES

1. V. I. Haltrin, "Self-consistent approach to the solution of the light transfer problem for irradiances in marine waters with arbitrary turbidity, depth and surface illumination" *Appl. Optics*, **37**, 3773-3784 (1998).
2. V. I. Haltrin (*a.k.a*. В. И. Халтурин), "Propagation of light in sea depth," in *Remote Sensing of the Sea and the Influence of the Atmosphere* (in Russian), V. A. Urdenko and G. Zimmermann, Eds., Academy of Sciences of the German Democratic Republic Institute for Space Research, Berlin, pp. 20-62, 1985.
3. V. I. Haltrin and G. W. Kattawar "Self-consistent solutions to the equation of transfer with elastic and inelastic scattering in oceanic optics: I. Model," *Appl. Optics*, **32**, pp. 5,356-5,367, 1993.
4. P. M. Morse and H. Feshbach, *Methods of Theoretical Physics*, **I**, McGraw Hill, New York-Toronto-London, pp. 122-123, 1953.
5. Fragments of sea water optical model may be found in Refs.[3, 17]. The final version is presented in Ref. [6].
6. V. I. Haltrin, "One-parameter model of seawater optical properties," in *Ocean Optics XIV*, (to be published by SPIE in 1999). See also: *Ocean Optics XIV CD-ROM*, Kailua-Kona, Hawaii, USA, November 10-13 1998, (Office of Naval Research, Ocean, Atmosphere, and Space S&T Department, November 1998).
7. V. I. Haltrin, "Apparent optical properties of the sea illuminated by Sun and sky: the optically deep sea," *Applied Optics*, 37, No. 36 (20 December 1998).
8. R. M. Pope, and E. S. Fry, "Absorption spectrum (380-700 nm) of pure water: II. Integrating cavity measurements," *Appl. Optics*, **36**, pp. 8,710-8,723, 1997.
9. L. Prieur, and S. Sathyendranath, "An Optical Classification of Coastal and oceanic waters based on the specific spectral absorption curves of phytoplankton pigments, dissolved organic matter, and other particulate materials," *Limnol. Oceanogr.*, **26**, pp. 671-689, 1981.
10. K. L. Carder, R. G. Stewart, G. R. Harvey, and P. B. Ortner, "Marine humic and fulvic acids: Their effects on remote sensing of ocean chlorophyll," *Limnol. Oceanogr*. **34**, pp. 68-81, 1989.
11. S. K. Hawes, K. L. Carder, and G. R. Harvey, "Quantum fluorescence efficiencies of fulvic and humic acids: effect on ocean color and fluorometric detection", in SPIE, **1750**, G. D. Gilbert, ed., *Ocean Optics XI*, pp. 212-223, 1992.
12. O. V. Kopelevich, "Small-Parametric Model of the Optical Properties of Seawater," in *Ocean Optics*, ***I**: Physical Ocean Optics*, A. S. Monin, ed., in Russian, Nauka Publishing, Moscow, pp. 208-234, 1983.
13. A. Morel, and L. Prieur, "Analysis of Variations in Ocean Color," *Limnol. Oceanogr.*, **22**, 709–722, 1977.
14. Some fragments of atmospheric optical model used here may be found in Refs.[7, 18]. The final version is presented in Ref.[15].
15. V. I. Haltrin, "Empirical relationship between aerosol scattering phase function and optical thickness of atmosphere above the ocean," in *Propagation and Imaging through the Atmosphere II, Proceedings of SPIE*, Vol. 3433, Luc R. Bissonette, ed., ISBN 0-8194-2888-4, (SPIE, Bellingham, WA, USA, 1998), pp. 334-342.
16. E. J. McCartney, *Optics of the Atmosphere*, Wiley, New York, 1976.
17. V. I. Haltrin, G. W. Kattawar and A. D. Weidemann, "Modeling of elastic and inelastic scattering effects in oceanic optics," in *Ocean Optics XIII: SPIE Proceedings*, **2963**, S. G. Ackleson and R. Frouin, eds., Bellingham, WA, pp. 597-602, 1997.
18. V. I. Haltrin (*a.k.a*. В. И. Халтурин), and V. A. Urdenko, "Propagation of Light in the Atmosphere and Remote Sensing of Optical Properties of Seawater," in *Optical Remote Sensing of the Sea and the Influence of the Atmosphere*, V. A. Urdenko and G. Zimmermann, Eds., in Russian, German Democratic Republic Academy of Sciences Institute for Space Research, Berlin-Moscow-Sevastopol, pp. 63-90, 1985.